

5. Linearna funkcija. Sustav jednačbi

Podsjetimo se osnovnih pravila;

Opcénito, ako su $a \neq 0$ i b zadani racionalni brojevi, onda je funkcija zadana formulom

$$f(x) = ax + b, \quad x \in \mathbb{Q}$$

zove LINEARNA (AFINA) FUNKCIJA

Ako je $b = 0$, linearna funkcija ima oblik $f(x) = ax$, a to je funkcija PROPORCIONALNOSTI.

PAZI: Svaka je proporcionalnost LINEARNA funkcija, ali sve linearne funkcije NISU PROPORCIONALNOSTI.

Graf funkcije

$$f(x) = ax + b \quad \text{je pravac } y = ax + b.$$

Parametri: a i b

a = parametar a nazivamo KOEFICIJENT smjera pravca

funkcija $f(x) = ax + b$ je RASTUĆA, ako je koeficijent smjera $a > 0$ (tj. pozitivan a)

– PADAJUĆA, ako je koeficijent smjera $a < 0$ (tj. a je negativan)

Parametar b – ima značenje ordinate sjecišta pravca $y = ax + b$ i ordinatne osi (y) i zato se naziva ODJSTACOM PRAVCA NA ORDINATNOJ OSI.

5.1. Graf linearne jednadžbe

1.

1. $x+y=8 \Rightarrow y=8-x$

Nul točke:

$$f(0)=8-x$$

x	0	1
y	8	7

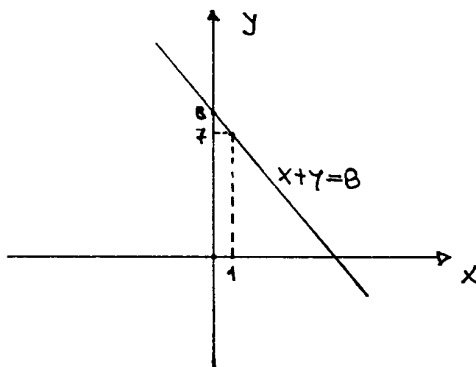
$$=8-0$$

$$=8$$

$$f(1)=8-x$$

$$=8-1$$

$$=7$$



2. $2x-y=0$
 $-y=-2x$
 $y=2x$

Nul točke

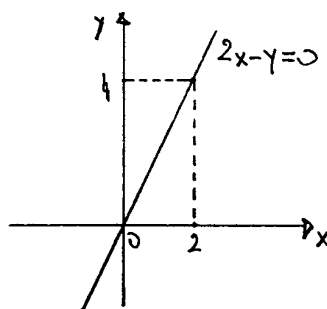
$$f(0)=2 \cdot 0$$

$$f(0)=0$$

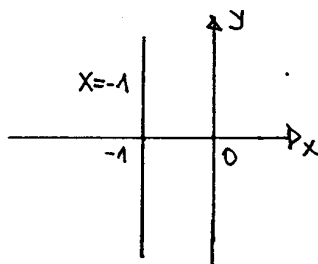
$$f(2)=2 \cdot 2$$

$$=4$$

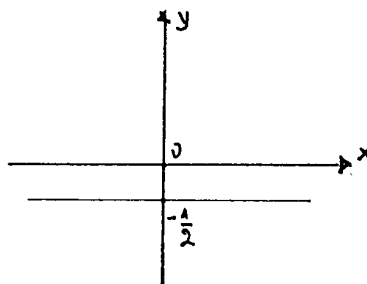
x	0	2
y	0	4



3. $x=-1$



4. $2y+1=0$
 $2y=-1 \div 2$
 $y=-\frac{1}{2}$



1)

5. $3x + 4y = 6$

$$4y = 6 - 3x \quad | :4$$

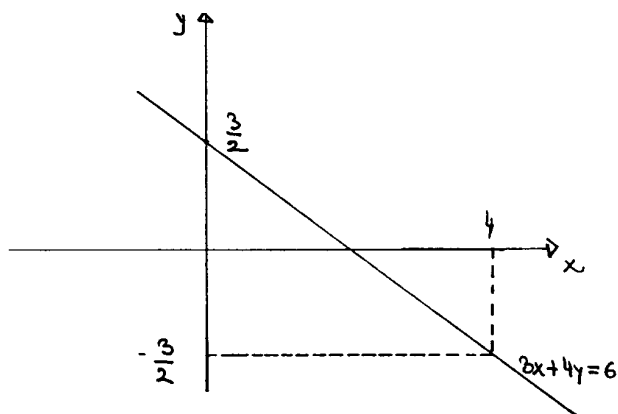
$$y = \frac{6}{4} - \frac{3}{4}x$$

$$y = \frac{3}{2} - \frac{3}{4}x$$

Nul točke:

$$f(0) = \frac{3}{2} - \frac{3}{4} \cdot 0$$
$$= \frac{3}{2}$$

$$\begin{array}{c|c|c} x & 0 & 4 \\ \hline y & \frac{3}{2} & -\frac{3}{2} \end{array}$$



$$f(4) = \frac{3}{2} - \frac{3}{4} \cdot 4$$

$$= \frac{3}{2} - 3 = \frac{3-6}{2} = -\frac{3}{2}$$

6. $x + y = 0$

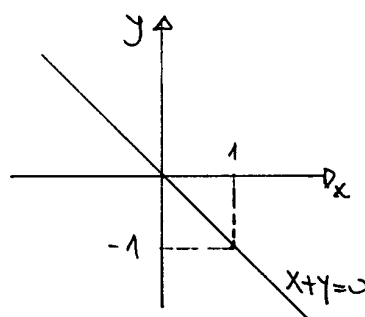
$$y = -x$$

Nul točke:

$$f(0) = 0$$

$$f(1) = -1$$

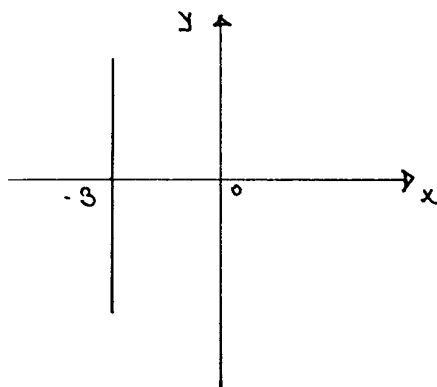
$$\begin{array}{c|c|c} x & 0 & 1 \\ \hline y & 0 & -1 \end{array}$$



7.

$$3x = -9 \quad | :3$$

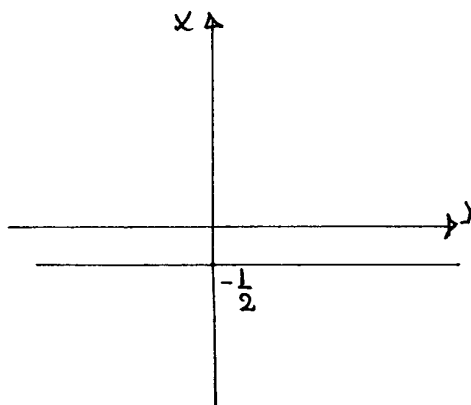
$$x = -3$$



8.

$$-2y = 1 \quad | :(-2)$$

$$y = -\frac{1}{2}$$



5.3. Graf funkcije $f(x) = |x|$

1.

1) $f(x) = 2|x|$

$\text{za: } x \geq 0$

$f(x) = 2x$

x	1	2
f(x)	2	4

$f(1) = 2 \cdot 1 = 2$

$f(2) = 2 \cdot 2 = 4$

$\text{za: } x < 0$

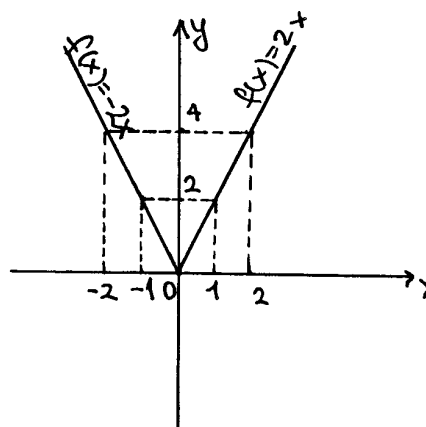
$f(x) = 2 \cdot (-x)$

$f(x) = -2x$

x	-1	-2
f(x)	2	4

$f(-1) = 2 \cdot (-(-1)) = 2$

$f(-2) = 2 \cdot (-(-2)) = 4$



2) $f(x) = \frac{1}{2}|x|$

$\text{za: } x \geq 0$

$f(x) = \frac{1}{2}x$

x	0	2
f(x)	0	1

$f(0) = \frac{1}{2} \cdot 0 = 0$

$f(2) = \frac{1}{2} \cdot 2 = 1$

$\text{za } x < 0$

$f(x) = \frac{1}{2}(-x)$

$f(x) = -\frac{1}{2}x$

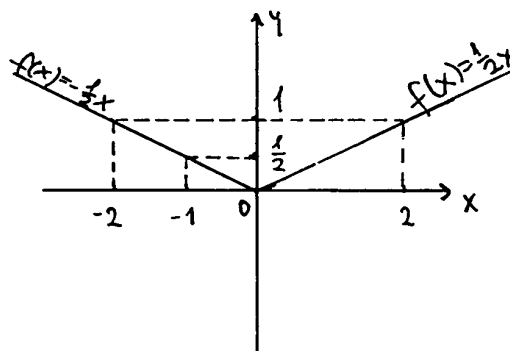
x	-1	-2
f(x)	$\frac{1}{2}$	1

$f(-1) = -\frac{1}{2} \cdot (-1) = \frac{1}{2}$

$f(-1) = \frac{1}{2}$

$f(-2) = -\frac{1}{2} \cdot (-2) = 1$

$f(-2) = 1$



3) $f(x) = 3|x|$

$\text{za: } x \geq 0$

$f(x) = 3x$

x	0	1
f(x)	0	3

$f(0) = 3 \cdot 0 = 0$

$f(1) = 3 \cdot 1 = 3$

$\text{za } x < 0$

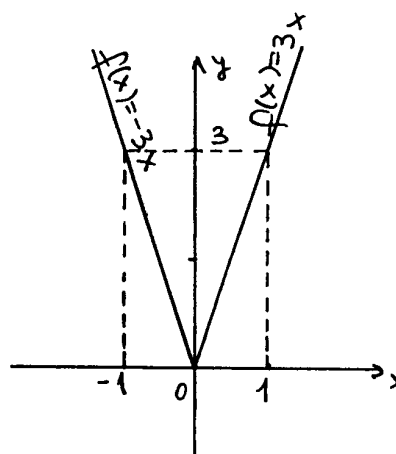
$f(x) = 3 \cdot (-x)$

$f(x) = -3x$

x	-1	0
f(x)	3	0

$f(-1) = -3 \cdot (-1) = 3$

$f(0) = -3 \cdot 0 = 0$



1) 4) $f(x) = -|x|$

$$\text{za: } x \geq 0$$

$$f(x) = -x$$

x	0	1
f(x)	0	-1

$$f(0) = -0 = 0$$

$$f(1) = -1$$

$$\text{za: } x < 0$$

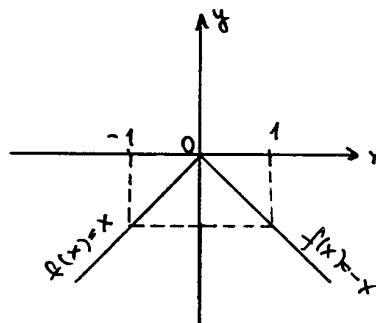
$$f(x) = -(-x)$$

$$f(x) = x$$

x	0	-1
f(x)	0	-1

$$f(0) = 0$$

$$f(-1) = -1$$



5) $f(x) = -\frac{1}{2}|x|$

$$\text{za: } x \geq 0$$

$$f(x) = -\frac{1}{2}x$$

x	0	2
f(x)	0	-1

$$f(0) = -\frac{1}{2} \cdot 0 = 0$$

$$f(2) = -\frac{1}{2} \cdot 2 = -1$$

$$\text{za: } x < 0$$

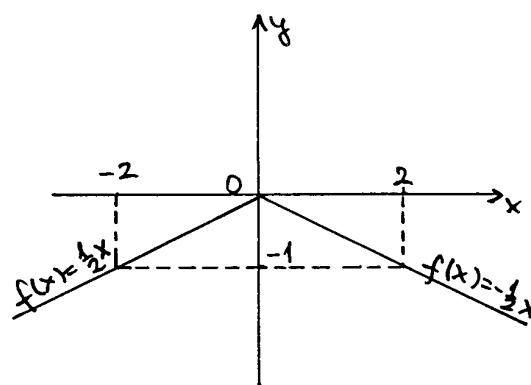
$$f(x) = -\frac{1}{2} \cdot (-x)$$

$$f(x) = \frac{1}{2}x$$

x	0	-2
f(x)	0	-1

$$f(0) = \frac{1}{2} \cdot 0 = 0$$

$$f(-2) = \frac{1}{2} \cdot -2 = -1$$



6) $f(x) = \frac{3}{4}|x|$

$$\text{za: } x \geq 0$$

$$f(x) = \frac{3}{4}x$$

x	0	4
f(x)	0	3

$$f(0) = \frac{3}{4} \cdot 0 = 0$$

$$f(4) = \frac{3}{4} \cdot 4 = 3$$

$$\text{za: } x < 0$$

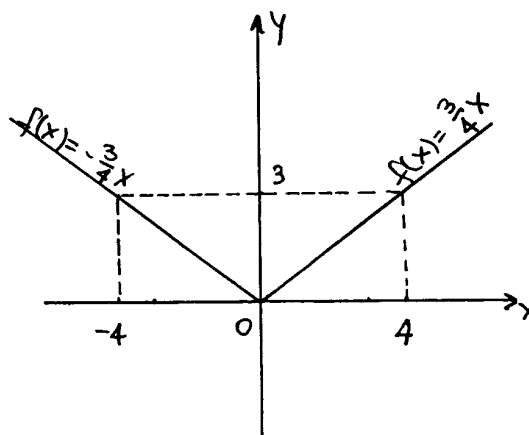
$$f(x) = \frac{3}{4} \cdot (-x)$$

$$f(x) = -\frac{3}{4}x$$

x	0	-4
f(x)	0	3

$$f(0) = -\frac{3}{4} \cdot 0 = 0$$

$$f(-4) = -\frac{3}{4} \cdot -4 = 3$$



5.4. Sjecište dvaju pravaca

1.

$$1) \quad \begin{array}{l} x-y=5 \rightarrow x=5+y \\ x-2y=2 \end{array}$$

$$\begin{array}{l} \boxed{\begin{array}{l} x=5+y \\ x-2y=2 \end{array}} \rightarrow \begin{array}{l} x=5+3 \\ x=8 \end{array} \\ \hline 5+y-2y=2 \\ 5-y=2 \\ -y=2-5 \\ -y=-3 \quad | \cdot (-1) \\ y=3 \end{array}$$

$$\underline{\underline{x=8, y=3}}$$

$$2) \quad \begin{array}{l} x+3y=5 \rightarrow x=5-3y \\ 2x-y=3 \end{array}$$

$$\begin{array}{l} 2 \cdot (5-3y) - y = 3 \\ 10 - 6y - y = 3 \\ -7y = 3 - 10 \\ -7y = -7 \quad | \cdot (-1) \\ y=1 \rightarrow \begin{array}{l} x=5-3y \\ x=5-3 \cdot 1 \\ x=5-3 \\ x=2 \end{array} \end{array}$$

$$\underline{\underline{x=2, y=1}}$$

$$3) \quad \begin{array}{l} x-2y+11=0 \longrightarrow -2y=x-11 \quad | :(-2) \\ -2x+4y+3=0 \longrightarrow 4y=2x-3 \quad | : (4) \end{array}$$

$$\begin{array}{l} x=2y-11 \\ -2x+4y+3=0 \\ -2(2y-11)+4y=-3 \\ -4y+22+4y=-3 \\ 22=-3 \\ \downarrow \\ \text{NETOČNA TURDNJA!} \end{array}$$

$$\begin{array}{l} y = \frac{1}{2}x + \frac{11}{2} \\ y = \frac{1}{2}x - \frac{3}{4} \end{array}$$

vidimo da su koeficijenti
smjera prvog i drugog
pravca jednaki, $(\frac{1}{2})$
što znači da su ti
pravci međusobno
paralelni.

$$4) \quad \begin{array}{l} 3x+2y=6 \rightarrow 3x=-2y+6 \quad | :3 \\ 2x-3y=4 \quad \quad \quad x=-\frac{2}{3}y+2 \end{array}$$

$$2 \cdot (-\frac{2}{3}y+2) - 3y = 4$$

$$-\frac{4}{3}y + 4 - 3y = 4 \quad | :3$$

$$-4y + 12 - 9y = 12$$

$$-13y = 12 - 12$$

$$-13y = 0 \quad | : (-13)$$

$$\underline{\underline{y=0}} \rightarrow \begin{array}{l} 2x-3y=4 \\ 2x-3 \cdot 0=4 \\ 2x=4 \quad | :2 \\ \underline{\underline{x=2}} \end{array}$$

$$\underline{\underline{x=2, y=0}}$$

$$9. \quad \begin{aligned} P_1 \dots & x - 4y - 6 = 0 \\ P_2 \dots & 2x - y + 9 = 0 \\ P_3 \dots & 3x + 2y - 4 = 0 \end{aligned}$$

vrh A: $P_2 \cap P_3$:

$$\begin{aligned} P_2 \dots & 2x - y + 9 = 0 \quad | \cdot 2 \\ P_3 \dots & 3x + 2y - 4 = 0 \\ \hline & 4x - 2y + 18 = 0 \\ & 3x + 2y - 4 = 0 \quad | + \\ \hline & 7x + 14 = 0 \\ & 7x = -14 \quad | : 7 \\ & x = -2 \end{aligned}$$

$$\begin{aligned} 2x - y + 9 &= 0 \\ 2 \cdot (-2) - y &= -9 \\ -4 - y &= -9 \\ -y &= -9 + 4 \\ -y &= -5 \\ y &= 5 \end{aligned}$$

A(-2, 5)

vrh B: $P_1 \cap P_3$

$$\begin{aligned} P_1 \dots & x - 4y - 6 = 0 \quad | \cdot (-3) \\ P_3 \dots & 3x + 2y - 4 = 0 \\ \hline & -3x + 12y + 18 = 0 \\ & 3x + 2y - 4 = 0 \quad | + \\ \hline & 14y = -14 \quad | : 14 \\ & y = -1 \end{aligned}$$

$$\begin{aligned} 3x + 2y - 4 &= 0 \\ 3x + 2 \cdot (-1) - 4 &= 0 \\ 3x - 2 - 4 &= 0 \\ 3x - 6 &= 0 \\ 3x &= 6 \quad | : 3 \\ x &= 2 \end{aligned}$$

B(2, -1)

vrh C:

$$\begin{aligned} P_1 \dots & x - 4y - 6 = 0 \quad | \cdot (-2) \\ P_2 \dots & 2x - y + 9 = 0 \\ \hline & -2x + 8y + 12 = 0 \\ & 2x - y + 9 = 0 \quad | + \\ \hline & 7y + 21 = 0 \\ & 7y = -21 \quad | : 7 \\ & y = -3 \end{aligned}$$

$$\begin{aligned} 2x - y + 9 &= 0 \\ 2x - (-3) + 9 &= 0 \\ 2x + 3 + 9 &= 0 \\ 2x + 12 &= 0 \\ 2x &= -12 \quad | : 2 \\ x &= -6 \end{aligned}$$

C(-6, -3)

površina trokuta:

$$P = \frac{1}{2} | x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) |$$

$$A(-2, 5); x_1 = -2, y_1 = 5$$

$$B(2, -1); x_2 = 2, y_2 = -1$$

$$C(-6, -3); x_3 = -6, y_3 = -3$$

$$P = \frac{1}{2} | (-2)(-1 - (-3)) + 2(-3 - 5) + (-6)(5 - (-1)) |$$

$$P = \frac{1}{2} | (-2) \cdot 2 + 2 \cdot (-8) + (-6) \cdot 6 |$$

$$P = \frac{1}{2} | -4 - 16 - 36 |$$

$$P = \frac{1}{2} | -56 |$$

$$P = \frac{56}{2}$$

P = 28

10.

jednadžbe srednjica trokuta:

$$P_1 \dots 2x - 3y - 4 = 0$$

$$P_2 \dots 3x - 2y - 1 = 0$$

$$P_3 \dots \underline{x + y - 2 = 0}$$

$$P_1 \dots y = \frac{2}{3}x - \frac{4}{3}$$

x	0	2	$y(0) = \frac{2}{3} \cdot 0 - \frac{4}{3} = -\frac{4}{3}$
y	$-\frac{4}{3}$	0	$y(2) = \frac{2}{3} \cdot 2 - \frac{4}{3} = 0$

$$P_2 \dots y = \frac{3}{2}x - \frac{1}{2}$$

x	0	2	$y(0) = \frac{3}{2} \cdot 0 - \frac{1}{2} = -\frac{1}{2}$
y	$-\frac{1}{2}$	$\frac{5}{2}$	$y(2) = \frac{3}{2} \cdot 2 - \frac{1}{2} = \frac{5}{2}$

$$P_3 \dots y = -x + 2$$

x	0	2	$y(0) = 2$
y	2	0	$y(2) = 0$

polovište AB:

$$P_2 \cap P_3$$

$$3x - 2y - 1 = 0$$

$$x + y - 2 = 0 \quad | \cdot 2$$

$$3x - 2y - 1 = 0$$

$$2x + 2y - 4 = 0 \quad | +$$

$$5x - 5 = 0$$

$$5x = 5 \quad | :5$$

$$x = 1$$

$$x + y - 2 = 0$$

$$1 + y - 2 = 0$$

$$y = 2 - 1$$

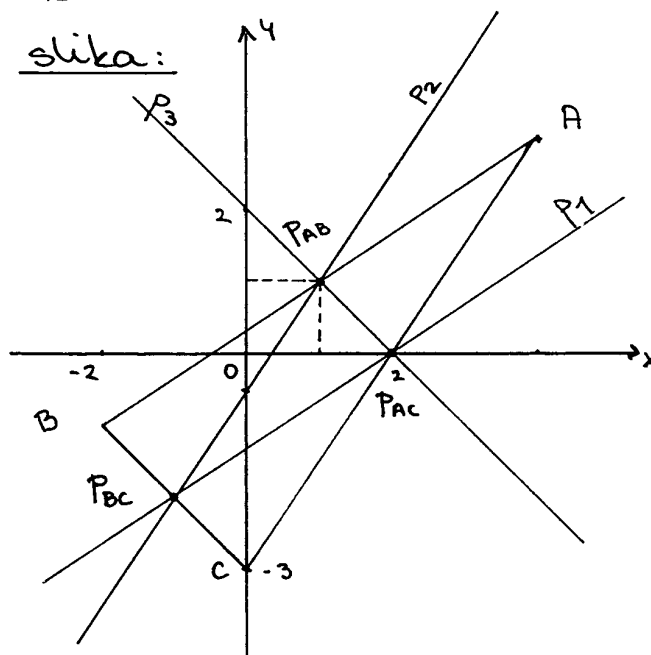
$$y = 1$$

$$\underline{\underline{P_{AB}(1, 1)}}$$

* Srednjice trokuta sjeku se na polovištima stranica.

znaci dvije srednjice se sjeku na polovici jedne stranice. Tražimo presjek jednadžbi P_1 i P_2 da dobijemo koordinate polovišta stranice a, koja se nalazi između vrhova B i C.

slika:



polovište AC:

$$P_1 \cap P_3$$

$$2x - 3y - 4 = 0$$

$$x + y - 2 = 0 \quad | \cdot 3$$

$$2x - 3y - 4 = 0$$

$$3x + 3y - 6 = 0 \quad | +$$

$$5x - 10 = 0$$

$$5x = 10 \quad | :5$$

$$x = 2$$

$$x + y - 2 = 0$$

$$2 + y - 2 = 0$$

$$y = 0$$

$$\underline{\underline{P_{AC}(2, 0)}}$$

polovište BC:

$$P_1 \cap P_2$$

$$2x - 3y - 4 = 0 \quad | \cdot 2$$

$$3x - 2y - 1 = 0 \quad | \cdot (-3)$$

$$4x - 6y - 8 = 0$$

$$-9x + 6y + 3 = 0 \quad | +$$

$$-5x - 5 = 0$$

$$-5x = 5 \quad | :(-5)$$

$$x = -1$$

$$2x - 3y - 4 = 0$$

$$2(-1) - 3y - 4 = 0$$

$$-3y = 6 \quad | :(-3)$$

$$y = -2$$

$$\underline{\underline{P_{BC}(-1, -2)}}$$

5.5. Sustav linearnih jednažbi

1.

$$1) \quad \begin{array}{r} 3x - 2y + 4 = 0 \\ 5x - 3y + 6 = 0 \end{array}$$

$$\begin{array}{r} 3x - 2y = -4 \quad / \cdot (-5) \\ 5x - 3y = -6 \quad / \cdot 3 \end{array}$$

$$\begin{array}{r} -15x + 10y = 20 \\ 15x - 9y = -18 \quad | + \end{array}$$

$$10y - 9y = 20 - 18$$

$$y = 2$$

$$3x - 2y = -4$$

$$3x - 2 \cdot 2 = -4$$

$$3x = -4 + 4$$

$$3x = 0$$

$$x = 0$$

$$\underline{x=0, \quad y=2}$$

$$2) \quad \begin{array}{r} 4x + 3y - 12 = 0 \\ 2x + 5y + 15 = 0 \end{array}$$

$$\begin{array}{r} 4x + 3y = 12 \\ 2x + 5y = -15 \quad / \cdot (-2) \end{array}$$

$$\begin{array}{r} 4x + 3y = 12 \\ -4x - 10y = 30 \quad | + \end{array}$$

$$3y - 10y = 12 + 30$$

$$-7y = 42 \quad / \cdot (-7)$$

$$y = -6$$

$$2x + 5y = -15$$

$$2x + 5(-6) = -15$$

$$2x - 30 = -15$$

$$2x = -15 + 30$$

$$2x = 15 \quad / \cdot 2$$

$$x = 7.5$$

$$\underline{x=7.5, \quad y=-6}$$

$$3) \quad \begin{array}{r} 6x - 5y = 3 \quad / \cdot 7 \\ 7x - 8y = 10 \quad / \cdot (-6) \end{array}$$

$$\begin{array}{r} 42x - 35y = 21 \\ -42x + 48y = -60 \quad | + \end{array}$$

$$13y = -39 \quad / \cdot 13$$

$$y = -3$$

$$6x - 5(-3) = 3$$

$$6x + 15 = 3$$

$$6x = 3 - 15$$

$$6x = -12 \quad / \cdot 6$$

$$x = -2$$

$$\underline{x=-2, \quad y=-3}$$

$$4) \quad \begin{array}{r} 0.3x - 0.5y + 0.9 = 0 \\ 2.1x + y - 7.2 = 0 \quad / \cdot 0.5 \end{array}$$

$$\begin{array}{r} 0.3x - 0.5y + 0.9 = 0 \\ 1.05x + 0.5y - 3.6 = 0 \quad | + \end{array}$$

$$1.35x - 2.7 = 0$$

$$1.35x = 2.7 \quad / \cdot 10$$

$$13.5x = 27 \quad / \cdot 13.5$$

$$x = 2$$

$$2.1x + y - 7.2 = 0$$

$$2.1 \cdot 2 + y = 7.2$$

$$4.2 + y = 7.2$$

$$y = 7.2 - 4.2$$

$$y = 3$$

$$\underline{x=2, \quad y=3}$$

$$1. \quad 5) \quad \begin{array}{l} 0.1x + 0.5y = 0.4 \\ \underline{0.2x + y = 0.2} \end{array} \quad \Rightarrow \quad y = 0.2 - 0.2x$$

sustav će mo riješiti metodom supstitucije

$$\cancel{0.2x} + 0.2 - \cancel{0.2x} = 0.2$$

Sustav nema rješenja

$$6.) \quad \begin{array}{l} 2.4x - 5.1y + 10.5 = 0 \quad | \cdot 10 \\ \underline{3x + 2y - 12 = 0} \end{array} \quad \Rightarrow \quad \begin{array}{l} 3x + 6 - 12 = 0 \\ 3x = 6 \\ x = 2 \end{array} \quad \begin{array}{l} \text{metoda suprotnih} \\ \text{koefficijenata} \end{array}$$

$$\begin{array}{l} 24x - 51y + 105 = 0 \\ \underline{3x + 2y - 12 = 0 \quad | \cdot (-8)} \\ 24x - 51y + 105 = 0 \\ \underline{-24x - 16y + 96 = 0} \\ -67y = -201 \\ y = 3 \end{array}$$

2.

$$1) \quad \begin{array}{l} 9x + 7y = 50 \quad | \cdot 5 \\ \underline{11x - 5y = 156 \quad | \cdot 7} \end{array} \quad \Rightarrow \quad \begin{array}{l} 11 \cdot 11 - 5y = 156 \\ -5y = 156 - 121 \\ -5y = 35 \\ y = -7 \end{array}$$

$$\begin{array}{l} 45x + 35y = 250 \\ \underline{77x - 35y = 1092} \\ 122x = 1342 \\ x = 11 \end{array}$$

$$2) \quad \begin{array}{l} 15x + 7y = 11 \quad | \cdot 9 \\ \underline{8x - 9y = -96 \quad | \cdot 7} \end{array} \quad \Rightarrow \quad \begin{array}{l} -24 - 9y = -96 \\ -9y = -96 + 24 \\ -9y = -72 \\ y = 8 \end{array}$$

metoda suprotnih koefficijenata

$$\begin{array}{l} 135x + 63y = 99 \\ \underline{56x - 63y = -672} \\ 191x = -573 \\ x = -3 \end{array}$$