Ovo su zadaci uz LIMESE – test zadaci br.7. , 8. , 9. ,10 .

9.

1) $a_n = 1 + \frac{1}{n} + \frac{1}{n^2}$

$a_n = \frac{n^2+n+1}{n^2}$

$\lim\limits_{n\to\infty} \frac{n^2+n+1}{n^2} \Big/ {:}n^2 = \lim\limits_{n\to\infty} \frac{1+\frac{1}{n}+\frac{1}{n^2}}{1} = \underline{\underline{1}}$

2) $a_n = \frac{1}{n^2} - \frac{2}{n} - 3 = \frac{1-2n-3n^2}{n^2}$

$\lim\limits_{n\to\infty} \frac{1-2n-3n^2}{n^2} \Big/ {:}n^2 = \lim\limits_{n\to\infty} \frac{\frac{1}{n^2}-\frac{2}{n}-3}{1} = -3 /\!/$

3) $a_n = \frac{3n^2-n+1}{5n^2+n-1}$

$\lim\limits_{n\to\infty} \frac{3n^2-n+1}{5n^2+n-1} \Big/ {:}n^2 = \lim\limits_{n\to\infty} \frac{3-\frac{1}{n}+\frac{1}{n^2}}{5+\frac{1}{n}-\frac{1}{n^2}} = \underline{\underline{\frac{3}{5}}}$

4) $a_n = \frac{n^2-n+1}{1+n-n^2}$

$\lim\limits_{n\to\infty} \frac{n^2-n+1}{1+n-n^2} \Big/ {:}n^2 = \lim\limits_{n\to\infty} \frac{1-\frac{1}{n}+\frac{1}{n^2}}{\frac{1}{n^2}+\frac{1}{n}-1} = \underline{\underline{-1}}$

5) $a_n = \frac{1-2n}{3n^2-n-2}$

$\lim\limits_{n\to\infty} \frac{1-2n}{3n^2-n-2} \Big/ {:}n^2 = \lim\limits_{n\to\infty} \frac{\frac{1}{n^2}-\frac{2}{n}}{3-\frac{1}{n}-\frac{2}{n^2}} = \underline{\underline{0}}$

6) $a_n = \frac{(n^2-n+1)(n^2-n-1)}{2n(1-n^3)} = \frac{(n^2-n)^2-1}{2n-2n^4} =$

$= \frac{n^4-2n^3-n^2-1}{2n-2n^4}$

$\lim\limits_{n\to\infty} \frac{n^4-2n^3-n^2-1}{2n-2n^4} \Big/ {:}n^4 = \lim\limits_{n\to\infty} \frac{1-\frac{2}{n}-\frac{1}{n^2}-\frac{1}{n^4}}{\frac{2}{n^3}-2} = \underline{\underline{-\frac{1}{2}}}$

7) $a_n = \frac{1-3n}{2+3{,}5n} \cdot \frac{2n^2-n+4}{3n-0{,}8n^2} = \frac{2n^2-n+4-6n^3+3n^2-12n}{6n-1{,}6n^2+10{,}5n^2-2{,}8n^3} =$

$a_n = \frac{-6n^3+5n^2-13n+4}{-2{,}8n^3+8{,}9n^2+6n}$

$\lim\limits_{n\to\infty} \frac{-6n^3+5n^2-13n+4 \quad /:n^3}{-2{,}8n^3+8{,}9n^2+6n \quad /:n^3} = \lim\limits_{n\to\infty} \frac{-6+\frac{5}{n}-\frac{13}{n^2}+\frac{4}{n^3}}{-2{,}8+\frac{8{,}9}{n}+\frac{6}{n^3}} =$

$= \frac{\cancel{-6}^{\,3}}{\cancel{-2{,}8}_{\,1{,}4}} = \frac{3}{1{,}4} = \frac{3}{\frac{14}{10}} = \frac{30}{14}$

8) $a_n = \frac{n(n+1)(n+2)}{(n+3)(n+4)(n+5)}$

$\lim\limits_{n\to\infty} \frac{n(n+1)(n+2) \quad /:n^3}{(n+3)(n+4)(n+5) \quad /:n^3} = \lim\limits_{n\to\infty} \frac{\frac{n}{n}\,\frac{n+1}{n}\,\frac{n+2}{n}}{\frac{n+3}{n}\,\frac{n+4}{n}\,\frac{n+5}{n}} =$

$= \lim\limits_{n\to\infty} \frac{1\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)}{\left(1+\frac{3}{n}\right)\left(1+\frac{4}{n}\right)\left(1+\frac{5}{n}\right)} = 1$

9) $a_n = \frac{n(n+1)}{(n+2)(n+3)(n+4)}$

$\lim\limits_{n\to\infty} \frac{n(n+1) \quad /:n^3}{(n+2)(n+3)(n+4) \quad /:n^3} = \lim\limits_{n\to\infty} \frac{\frac{n}{n^2}\,\frac{n+1}{n}}{\frac{n+2}{n}\cdot\frac{n+3}{n}\,\frac{n+4}{n}} =$

$= \lim\limits_{n\to\infty} \frac{\frac{1}{n}\left(1+\frac{1}{n}\right)}{\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\left(1+\frac{4}{n}\right)} = \frac{0}{1} = 0$

10) $a_n = \frac{n!}{(n+1)!} = \frac{\cancel{1\cdot 2 \cdot \ldots \cdot n}}{\cancel{1\cdot 2 \cdot \ldots \cdot n}\cdot(n+1)} = \frac{1}{n+1}$

$\lim\limits_{n\to\infty} \frac{1}{n+1} = 0$

10.

1) $\lim\limits_{n\to\infty} \frac{2n+3}{3n-1} = \lim\limits_{n\to\infty} \frac{2+\frac{3}{n}}{3-\frac{1}{n}} = \frac{2}{3}$

2) $\lim\limits_{n\to\infty} \frac{3n^2+5n+1}{2n^2-n+5} = \lim\limits_{n\to\infty} \frac{3+\frac{5}{n}+\frac{1}{n^2}}{2-\frac{1}{n}+\frac{5}{n^2}} = \frac{3}{2}$

3) $\lim\limits_{n\to\infty} \frac{\sqrt{n+1}+n}{\sqrt{n^2+1}+\sqrt{n}} = \lim\limits_{n\to\infty} \frac{\sqrt{\frac{1}{n}+\frac{1}{n^2}}+1}{\sqrt{1+\frac{1}{n^2}}+\sqrt{\frac{1}{n}}} = \frac{1}{\sqrt{1}} = 1$

4) $\lim\limits_{n\to\infty} \frac{\sqrt{3n^3-2n}+n}{n^2+n} = \lim\limits_{n\to\infty} \frac{\sqrt{\frac{3}{n}-\frac{2}{n^3}}+\frac{1}{n}}{1+\frac{1}{n}} = \frac{0}{1} = 0$

5) $\lim\limits_{n\to\infty} \frac{2^n+3^n+0{,}5^n}{2^n-3^n+0{,}5^n} = \lim\limits_{n\to\infty} \frac{\left(\frac{2}{3}\right)^n+1+\left(\frac{0{,}5}{3}\right)^n}{\left(\frac{2}{3}\right)^n-1+\left(\frac{0{,}5}{3}\right)^n} = \frac{1}{-1} = -1$ jer $\lim\limits_{n\to\infty}\left(\frac{a}{3}\right)^n = 0$

ako $a<3$

6) $\lim\limits_{n\to\infty} \frac{2^{n-2}}{2^n-2} = \lim\limits_{n\to\infty} \frac{1}{2^{n-(n-2)}-2^{1-(n-2)}} = \lim\limits_{n\to\infty} \frac{1}{4-2^{3-n}} = \frac{1}{4}$

11.

1) $a_n = \frac{\sqrt{n+1}}{\sqrt{n+2}}$

$\lim\limits_{n\to\infty} \frac{\sqrt{n+1}\ /:\sqrt{n}}{\sqrt{n+2}\ /:\sqrt{n}} = \lim\limits_{n\to\infty} \frac{\sqrt{\frac{n}{n}+\frac{1}{n}}}{\sqrt{\frac{n}{n}+\frac{2}{n}}} = \underline{\underline{1}}$

2) $a_n = \frac{\sqrt{n+1}}{\sqrt{n}+1}$

$\lim\limits_{n\to\infty} \frac{\sqrt{n+1}\ /:\sqrt{n}}{\sqrt{n}+1\ /:\sqrt{n}} = \lim\limits_{n\to\infty} \frac{\sqrt{1+\frac{1}{n}}}{1+\frac{1}{\sqrt{n}}} = \underline{\underline{1}}$

3) $a_n = \frac{\sqrt{n^2+1}}{n}$

$\lim\limits_{n\to\infty} \frac{\sqrt{n^2+1}\ /:n}{n\ /:n} = \lim\limits_{n\to\infty} \frac{\sqrt{\frac{n^2+1}{n^2}}}{\frac{n}{n}} = \lim\limits_{n\to\infty} \frac{\sqrt{1+\frac{1}{n^2}}}{1} = \underline{\underline{1}}$

4) $a_n = \dfrac{\sqrt[3]{2n^3+1}}{\sqrt{2n^2-1}}$

$$\lim_{n\to\infty} \frac{\sqrt[3]{2n^3+1}\ /{:}n}{\sqrt{2n^2-1}\ /{:}n} = \lim_{n\to\infty} \frac{\sqrt[3]{\frac{2n^3+1}{n^3}}}{\sqrt{\frac{2n^2-1}{n^2}}} = \lim_{n\to\infty} \frac{\sqrt[3]{2+\frac{1}{n^3}}}{\sqrt[2]{2-\frac{1}{n^2}}} = \frac{\sqrt[3]{2}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} =$$

$$= \frac{\sqrt[3]{2}\cdot\sqrt{2}}{2} = \frac{\sqrt[6]{2^2\cdot 2^3}}{2} = \frac{\sqrt[6]{2^5}}{2}$$

5) $a_n = \sqrt{n+1} - \sqrt{n}$

$$\lim_{n\to\infty} (\sqrt{n+1} - \sqrt{n}) \cdot \frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} = \lim_{n\to\infty} \frac{\cancel{n}+1-\cancel{n}}{\sqrt{n+1}+\sqrt{n}} = 0$$

6) $a_n = \sqrt{n^2+n} - n$

$$\lim_{n\to\infty} (\sqrt{n^2+n}-n) \cdot \frac{\sqrt{n^2+n}+n}{\sqrt{n^2+n}+n} = \lim_{n\to\infty} \frac{\cancel{n^2}+n-\cancel{n^2}}{\sqrt{n^2+n}+n} =$$

$$= \lim_{n\to\infty} \frac{n\ /{:}n}{\sqrt{n^2+n}+n\ /{:}n} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n}}+1} = \frac{1}{2}$$

7) $a_n = \dfrac{\sqrt[3]{n^2+n}}{n+2}$

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^2+n}\ /{:}n}{n+2\ /{:}n} = \lim_{n\to\infty} \frac{\sqrt[3]{\frac{n^2+n}{n^3}}}{1+\frac{2}{n}} = \lim_{n\to\infty} \frac{\sqrt[3]{\frac{1}{n}+\frac{1}{n^2}}}{1+\frac{2}{n}} = \frac{0}{1} = 0$$

8) $a_n = \sqrt[3]{n} - \sqrt[3]{n+1}$

$$\lim_{n\to\infty} \left(\sqrt[3]{n} - \sqrt[3]{n+1}\right) \cdot \frac{\sqrt[3]{n^2} + \sqrt[3]{n(n+1)} + \sqrt[3]{(n+1)^2}}{\sqrt[3]{n^2} + \sqrt[3]{n(n+1)} + \sqrt[3]{(n+1)^2}} =$$

$$= \lim_{n\to\infty} \frac{n-(n+1)}{\sqrt[3]{n^2} + \sqrt[3]{n^2+n} + \sqrt[3]{(n+1)^2}} =$$

$$= \lim_{n\to\infty} \frac{\cancel{n}-\cancel{n}-1}{\sqrt[3]{n^2} + \sqrt[3]{n^2+n} + \sqrt[3]{(n+1)^2}} = \left(\frac{-1}{\infty}\right) = 0$$

9) $a_n = \frac{\sqrt{n}-2}{n+\sqrt{n}+1}$

$$\lim_{n\to\infty} \frac{\sqrt{n}-2}{n+\sqrt{n}+1} \begin{matrix} /:n \\ /:n \end{matrix} = \lim_{n\to\infty} \frac{\frac{\sqrt{n}}{n}-\frac{2}{n}}{1+\frac{\sqrt{n}}{n}+\frac{1}{n}} = \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n}}-\frac{2}{n}}{1+\frac{1}{\sqrt{n}}+\frac{1}{n}} = \frac{0}{1} = 0$$

10) $a_n = \sqrt{n^2+5n+1} - \sqrt{n^2-n}$

$$\lim_{n\to\infty} \left(\sqrt{n^2+5n+1} - \sqrt{n^2-n}\right) \cdot \frac{\sqrt{n^2+5n+1}+\sqrt{n^2-n}}{\sqrt{n^2+5n+1}+\sqrt{n^2-n}} =$$

$$= \lim_{n\to\infty} \frac{\cancel{n^2}+5n+1-\cancel{n^2}+n}{\sqrt{n^2+5n+1}+\sqrt{n^2-n}} = \lim_{n\to\infty} \frac{6n+1}{\sqrt{n^2+5n+1}+\sqrt{n^2-n}} \begin{matrix} /:n \\ /:n \end{matrix} =$$

$$= \lim_{n\to\infty} \frac{6+\frac{1}{n}}{\sqrt{1+\frac{5}{n}+\frac{1}{n^2}}+\sqrt{1-\frac{1}{n}}} = \frac{6}{\sqrt{1}+\sqrt{1}} = \frac{6}{2} = 3$$

12.

1) $a_n = \frac{\sqrt{n+2}}{\sqrt{n+1}+\sqrt{n+2}}$

$$\lim_{n\to\infty} \frac{\sqrt{n+2}}{\sqrt{n+1}+\sqrt{n+2}} \begin{matrix} /:\sqrt{n} \\ /:\sqrt{n} \end{matrix} = \lim_{n\to\infty} \frac{\sqrt{1+\frac{2}{n}}}{\sqrt{1+\frac{1}{n}}+\sqrt{1+\frac{2}{n}}} = \frac{1}{2}$$

2) $a_n = \frac{\sqrt{n+1}-n}{\sqrt{n+1}+n}$

$$\lim_{n\to\infty} \frac{\sqrt{n+1}-n}{\sqrt{n+1}+n} \begin{matrix} /:n \\ /:n \end{matrix} = \lim_{n\to\infty} \frac{\sqrt{\frac{n+1}{n^2}}-1}{\sqrt{\frac{n+1}{n^2}}+1} = \lim_{n\to\infty} \frac{\sqrt{\frac{1}{n}+\frac{1}{n^2}}-1}{\sqrt{\frac{1}{n}+\frac{1}{n^2}}+1} = -1$$

3) $a_n = \sqrt[3]{n^3+n^2+1} - \sqrt[3]{n^3-n^2+1}$

$$\lim_{n\to\infty} \left(\sqrt[3]{n^3+n^2+1} - \sqrt[3]{n^3-n^2+1}\right) \cdot \frac{\sqrt[3]{(n^3+n^2+1)^2}+\sqrt[3]{(n^3+n^2+1)(n^3-n^2+1)}+\sqrt[3]{(n^3-n^2}}{\sqrt[3]{(n^3+n^2+1)^2}+\sqrt[3]{(n^3+n+1)(n^3-n^2+1)}+\sqrt[3]{(n^3-n^2+1)^2}}$$

$$= \lim_{n\to\infty} \frac{n^3+n^2+1-(n^3-n^2+1)}{\sqrt[3]{(n^3+n+1)^2}+\sqrt[3]{(n^3+n+1)(n^3-n^2+1)}+\sqrt[3]{(n^3-n^2+1)^2}}$$

$$= \lim_{n\to\infty} \frac{\cancel{n^3}+n^2+\cancel{1}-\cancel{n^3}+n^2-\cancel{1}}{\sqrt[3]{(n^3+n+1)^2}+\sqrt[3]{(n^3+1)^2-(n^2)^2}+\sqrt[3]{(n^3-n^2+1)^2}}$$

$$= \lim_{n\to\infty} \frac{2n^2 \quad /:n^2}{\sqrt[3]{(n^3+n+1)^2}+\sqrt[3]{(n^3+1)^2-(n^2)^2}+\sqrt[3]{(n^3-n^2+1)^2} \quad /:n^2} =$$

$$= \lim_{n\to\infty} \frac{2}{\sqrt[3]{\frac{(n^3+n+1)^2}{n^6}}+\sqrt[3]{\frac{(n^3+1)^2}{n^6}-\frac{n^4}{n^6}}+\sqrt[3]{\frac{(n^3-n^2+1)^2}{n^6}}}$$

$$= \lim_{n\to\infty} \frac{2}{\sqrt[3]{\left(\frac{n^3+n^2+1}{n^3}\right)^2}+\sqrt[3]{\left(\frac{n^3+1}{n^3}\right)^2-\frac{1}{n^2}}+\sqrt[3]{\left(\frac{n^3-n^2+1}{n^3}\right)^2}}$$

$$= \lim_{n\to\infty} \frac{2}{\sqrt[3]{\left(1+\frac{1}{n}+\frac{1}{n^3}\right)^2}+\sqrt[3]{\left(1+\frac{1}{n^3}\right)^2-\frac{1}{n^2}}+\sqrt[3]{\left(1-\frac{1}{n}+\frac{1}{n^3}\right)^2}}$$

$$= \frac{2}{\sqrt[3]{1}+\sqrt[3]{1}+\sqrt[3]{1}} = \underline{\underline{\frac{2}{3}}}$$

4) $a_n = \sqrt{n+\sqrt{n+\sqrt{n}}} - \sqrt{n}$

$$\lim_{n\to\infty}\left(\sqrt{n+\sqrt{n+\sqrt{n}}}-\sqrt{n}\right)\cdot\frac{\sqrt{n+\sqrt{n+\sqrt{n}}}+\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}+\sqrt{n}} =$$

$$= \lim_{n\to\infty} \frac{\left(\sqrt{n+\sqrt{n+\sqrt{n}}}\right)^{\cancel{2}}-\left(\sqrt{n}\right)^{\cancel{2}}}{\sqrt{n+\sqrt{n+\sqrt{n}}}+\sqrt{n}} = \lim_{n\to\infty} \frac{\cancel{n}+\sqrt{n+\sqrt{n}}-\cancel{n} \quad /:\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}+\sqrt{n} \quad /:\sqrt{n}} =$$

$$= \lim_{n\to\infty} \frac{\sqrt{\frac{n+\sqrt{n}}{n}}}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}+\frac{\sqrt{n}}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\sqrt{1+\frac{1}{\sqrt{n}}}}{\sqrt{1+\sqrt{\frac{n+\sqrt{n}}{n^2}}}+1} = \underline{\underline{\frac{1}{2}}}$$

5) $a_n = \dfrac{3}{\sqrt{n+3}-\sqrt{n}} - \dfrac{1}{\sqrt{n+2}-\sqrt{n+1}}$

$$\lim_{n\to\infty}\left(\frac{3}{\sqrt{n+3}-\sqrt{n}}-\frac{1}{\sqrt{n+2}-\sqrt{n+1}}\right) = \lim_{n\to\infty}\frac{3\sqrt{n+2}-3\sqrt{n+1}-\sqrt{n+3}+\sqrt{n} \quad /:n}{\left(\sqrt{n+3}-\sqrt{n}\right)\left(\sqrt{n+2}-\sqrt{n+1}\right) \quad /:n} =$$

$$= \lim_{n\to\infty} \frac{3\sqrt{\frac{1}{n}+\frac{2}{n^2}}-3\sqrt{\frac{1}{n}+\frac{1}{n^2}}-\sqrt{\frac{1}{n}+\frac{3}{n^2}}+\sqrt{\frac{n}{n^2}}}{\left(\sqrt{\frac{n+3}{n}}-\sqrt{\frac{n}{n}}\right)\left(\sqrt{\frac{n+2}{n}}-\sqrt{\frac{n+1}{n}}\right)} = \underline{\underline{0}}$$

13. 1) $a_n = \dfrac{3^n+2^n}{3^n-2^n}$

$$\lim_{n\to\infty} \frac{3^n+2^n /:3^n}{3^n-2^n /:3^n} = \lim_{n\to\infty} \frac{1+\left(\frac{2}{3}\right)^n}{1-\left(\frac{2}{3}\right)^n} = 1$$

$\left(\frac{2}{3}\right)^n \to 0$ jer $\frac{2}{3}<1$

2) $a_n = \dfrac{3^{n-1}}{3^n-2}$

$$\lim_{n\to\infty} \frac{3^{n-1} /:3^n}{3^n-2 /:3^n} = \lim_{n\to\infty} \frac{\frac{3^{n-1}}{3^n}}{1-\frac{2}{3^n}} = \lim_{n\to\infty} \frac{\frac{1}{3}}{1-\frac{2}{3^n}} = \lim_{n\to\infty} \frac{1}{3-\frac{6}{3^n}} = \frac{1}{3}$$

$\frac{6}{3^n} \to 0$

3) $a_n = \dfrac{2^n-3^n}{4^n-6^n+9^n}$

$$\lim_{n\to\infty} \frac{2^n-3^n}{4^n-6^n+9^n} = \lim_{n\to\infty} \frac{2^n-3^n}{2^{2n}-(2\cdot 3)^n+3^{2n}} \cdot \frac{2^n+3^n}{2^n+3^n} =$$

$$= \lim_{n\to\infty} \frac{2^{2n}-3^{2n} /:3^{3n}}{2^{3n}+3^{3n} /:3^{3n}} = \lim_{n\to\infty} \frac{\frac{2^{2n}}{3^{3n}}-\frac{3^{2n}}{3^{3n}}}{\frac{2^{3n}}{3^{3n}}+1} = \lim_{n\to\infty} \frac{\frac{4^n}{27^n}-\frac{1}{3^n}}{\left(\frac{2}{3}\right)^n+1} =$$

$$= \lim_{n\to\infty} \frac{\left(\frac{4}{27}\right)^n - \frac{1}{3^n}}{\left(\frac{2}{3}\right)^n+1} = 0$$

$\left(\frac{4}{27}\right)^n \to 0$, $\frac{1}{3^n} \to 0$, $\left(\frac{2}{3}\right)^n \to 0$

4) $a_n = \dfrac{2^{1/n}-1}{2^{1/n}+1}$

$$\lim_{n\to\infty} \frac{2^{1/n}-1}{2^{1/n}+1} \cdot \frac{2^{n-1/n}+2^{n-2/n}+\dots+1}{2^{n-1/n}+2^{n-2/n}+\dots+1} =$$

$$= \lim_{n\to\infty} \frac{2-1}{\left(2^{1/n}+1\right)\left(2^{n-1/n}+2^{n-2/n}+\dots+1\right)} = 0$$

5) $a_n = \sqrt[n]{1+2^n}$

$$\lim_{n\to\infty} \sqrt[n]{1+2^n} \cdot \frac{\sqrt[n]{(1+2^n)^{n-1}}}{\sqrt[n]{(1+2^n)^{n-1}}} = \lim_{n\to\infty} \frac{1+2^n \quad /:2^n}{\sqrt[n]{(1+2^n)^{n-1}} \quad /:2^n} =$$

$$= \lim_{n\to\infty} \frac{\frac{1}{2^n}+1}{\sqrt[n]{\frac{(1+2^n)^{n-1}}{2^{2n}}}} = \lim_{n\to\infty} \frac{\frac{1}{2^n}+1}{\sqrt[n]{\frac{(1+2^n)^{n-1}}{2^2\cdot 2^{2n-2}}}} =$$

$$= \lim_{n\to\infty} \frac{\frac{1}{2^n}+1}{\sqrt[n]{\left(\frac{1+2^n}{2^n}\right)^{n-1} \cdot \frac{1}{4\cdot 2^{n-1}}}} = \lim_{n\to\infty} \frac{\frac{1}{2^n}+1}{\sqrt[n]{\left(1+\frac{1}{2^n}\right)^{n-1} \cdot \frac{1}{4\cdot 2^{n-1}}}} =$$

$$= \lim_{n\to\infty} \frac{\frac{1}{2^n}+1}{\sqrt[n]{\left(1+\frac{1}{2^n}\right)^{n-1}} \cdot \sqrt[n]{\frac{1}{2^n}\cdot\frac{1}{2}}} = \lim_{n\to\infty} \frac{\frac{1}{2^n}+1}{\sqrt[n]{\left(1+\frac{1}{2^n}\right)^{n-1}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt[n]{2}}} =$$

$$= \frac{1}{\frac{1}{2}} = 2$$

6) $a_n = \frac{2^n}{1+3^n}$

$$\lim_{n\to\infty} \frac{2^n \quad /:3^n}{1+3^n \quad /:3^n} = \lim_{n\to\infty} \frac{\left(\frac{2}{3}\right)^n}{\frac{1}{3^n}+1} = 0$$

7) $a_n = \frac{2^n+3^n}{n^2+2\cdot 3^n}$

$$\lim_{n\to\infty} \frac{2^n+3^n \quad /:3^n}{n^2+2\cdot 3^n \quad /:3^n} = \lim_{n\to\infty} \frac{\left(\frac{2}{3}\right)^n+1}{\frac{n^2}{3^n}+2} = \frac{1}{2}$$

8) $a_n = \frac{2\cdot 3^n+1}{2-5\cdot 3^n}$

$$\lim_{n\to\infty} \frac{2\cdot 3^n+1 \quad /:3^n}{2-5\cdot 3^n \quad /:3^n} = \lim_{n\to\infty} \frac{2+\frac{1}{3^n}}{\frac{2}{3^n}-5} = -\frac{2}{5}$$

9) $a_n = \left(\frac{1}{2}\right)^n \cdot \left(\frac{2}{3}\right)^n \cdot \left(\frac{3}{4}\right)^n = \left(\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{3}} \cdot \frac{\cancel{3}}{4}\right)^n = \left(\frac{1}{4}\right)^n = \frac{1}{4^n}$

$$\lim_{n\to\infty} \frac{1}{4^n} = 0$$

10) $a_n = \frac{3^n+4^n+5^n}{2^n+6^n}$

$$\lim_{n\to\infty} \frac{3^n+4^n+5^n \quad /:6^n}{2^n+6^n \quad /:6^n} = \lim_{n\to\infty} \frac{\left(\frac{3}{6}\right)^n+\left(\frac{4}{6}\right)^n+\left(\frac{5}{6}\right)^n}{\left(\frac{2}{6}\right)^n+1} = 0$$

2.6. Geometrijski red

1.

1) $2, 1, \frac{1}{2}, \ldots$

$a_1 = 2$

$q = \frac{1}{2}$

$$S_n = \frac{2}{1-\frac{1}{2}} = \frac{2}{\frac{1}{2}}$$

$$\boxed{S_n = 4}$$

Suma članova beskonačnog konvergentnog geometrijskog reda

$$S_n = \frac{a_1}{1-q}$$

2) $4, 2\frac{2}{3}, 1\frac{7}{9}, \ldots$

$a_1 = 4$

$$q = \frac{\frac{8}{3}}{4}$$

$q = \frac{2}{3}$

$$S_n = \frac{4}{1-\frac{2}{3}} =$$

$$S_n = \frac{4}{\frac{1}{3}}$$

$$\boxed{S_n = 12}$$

3) $1, \frac{1}{1.2}, \frac{1}{(1.2)^2}, \ldots$

$a_1 = 1$

$a_2 = \frac{1}{1,2}$

$$q = \frac{a_2}{a_1} = \frac{\frac{1}{1,2}}{1} = \frac{1}{1,2}$$

$$S_n = \frac{1}{1-\frac{1}{1,2}} = \frac{1}{\frac{1,2-1}{1,2}} =$$

$$S_n = \frac{1}{\frac{0,2}{1,2}} = 6$$

$$\boxed{S_n = 6}$$

4) $\sqrt{2}, \sqrt{\frac{1}{2}}, \frac{1}{4}\sqrt{2}, \ldots$

$a_1 = \sqrt{2}$

$$q = \frac{\sqrt{\frac{1}{2}}}{\sqrt{2}} = \frac{\frac{1}{\sqrt{2}}}{\sqrt{2}} = \frac{1}{2}$$

$$S_n = \frac{\sqrt{2}}{1-\frac{1}{2}} = \frac{\sqrt{2}}{\frac{1}{2}}$$

$$\boxed{S_n = 2\sqrt{2}}$$

5) $\frac{\sqrt{2}+1}{\sqrt{2}-1}, \frac{1}{2-\sqrt{2}}, \frac{1}{2}, \ldots$

$$a_1 = \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$q = \frac{\frac{1}{2-\sqrt{2}}}{\frac{\sqrt{2}+1}{\sqrt{2}-1}} = \frac{\frac{1}{\sqrt{2}(\cancel{\sqrt{2}-1})}}{\frac{\sqrt{2}+1}{\cancel{\sqrt{2}-1}}} =$$

$$q = \frac{1}{\sqrt{2}(\sqrt{2}+1)} = \frac{1}{2+\sqrt{2}}$$

$$S_n = \frac{\frac{\sqrt{2}+1}{\sqrt{2}-1}}{1-\frac{1}{2+\sqrt{2}}} = \frac{\frac{\sqrt{2}+1}{\sqrt{2}-1}}{\frac{\sqrt{2}+2-1}{2+\sqrt{2}}} = \frac{\frac{\cancel{\sqrt{2}+1}}{\sqrt{2}-1}}{\frac{\cancel{\sqrt{2}+1}}{2+\sqrt{2}}} =$$

$$= \frac{\sqrt{2}(\sqrt{2}+1)}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}(2+2\sqrt{2}+1)}{2-1} =$$

$$= \sqrt{2}(3+2\sqrt{2}) = \underline{\underline{3\sqrt{2}+4}}$$

$$\boxed{S_n = 3\sqrt{2}+4}$$

6) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$, 1, $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

$$a_1 = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$q = \frac{1}{\frac{\sqrt{3}+1}{\sqrt{3}-1}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$S_n = \frac{a_1}{1-q}$$

$$S_n = \frac{\frac{\sqrt{3}+1}{\sqrt{3}-1}}{1-\frac{\sqrt{3}-1}{\sqrt{3}+1}} = \frac{\frac{\sqrt{3}+1}{\sqrt{3}-1}}{\frac{\cancel{\sqrt{3}}+1-\cancel{\sqrt{3}}+1}{\sqrt{3}+1}} =$$

$$S_n = \frac{\frac{\sqrt{3}+1}{\sqrt{3}-1}}{\frac{2}{\sqrt{3}+1}} = \frac{(\sqrt{3}+1)^2}{2(\sqrt{3}-1)} =$$

$$S_n = \frac{3+2\sqrt{3}+1}{2(\sqrt{3}-1)} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1} =$$

$$S_n = \frac{(4+2\sqrt{3})(\sqrt{3}+1)}{2(3-1)} = \frac{\cancel{2}(2+\sqrt{3})(\sqrt{3}+1)}{\cancel{2}\cdot 2} =$$

$$S_n = \frac{2\sqrt{3}+2+3+\sqrt{3}}{2} = \frac{3\sqrt{3}+5}{2}$$

$$\boxed{S_n = \frac{3\sqrt{3}+5}{2}}$$

7) $2+\sqrt{2},\ 1+\sqrt{2},\ 1+\frac{\sqrt{2}}{2}$

$a_1=2+\sqrt{2}$

$q=\frac{1+\sqrt{2}}{2+\sqrt{2}}$

$S_n=\frac{a_1}{1-q}$

$$S_n=\frac{2+\sqrt{2}}{1-\frac{1+\sqrt{2}}{2+\sqrt{2}}}=\frac{2+\sqrt{2}}{\frac{2+\cancel{\sqrt{2}}-1-\cancel{\sqrt{2}}}{2+\sqrt{2}}}=\frac{(2+\sqrt{2})^2}{1}=4+4\sqrt{2}+2=6+4\sqrt{2}$$

$\boxed{S_n=6+4\sqrt{2}}$

8) $\frac{2+\sqrt{2}}{2-\sqrt{2}},\ 1,\ \frac{2-\sqrt{2}}{2+\sqrt{2}}$

$a_1=\frac{2+\sqrt{2}}{2-\sqrt{2}}$, $q=\frac{1}{\frac{2+\sqrt{2}}{2-\sqrt{2}}}=\frac{2-\sqrt{2}}{2+\sqrt{2}}$

$S_n=\frac{a_1}{1-q}$

$$S_n=\frac{\frac{2+\sqrt{2}}{2-\sqrt{2}}}{1-\frac{2-\sqrt{2}}{2+\sqrt{2}}}=\frac{\frac{2+\sqrt{2}}{2-\sqrt{2}}}{\frac{\cancel{2}+\sqrt{2}-\cancel{2}+\sqrt{2}}{2+\sqrt{2}}}=\frac{\frac{2+\sqrt{2}}{2-\sqrt{2}}}{\frac{2\sqrt{2}}{2+\sqrt{2}}}=\frac{(2+\sqrt{2})^2}{2\sqrt{2}(2-\sqrt{2})}\cdot\frac{2+\sqrt{2}}{2+\sqrt{2}}=$$

$$S_n=\frac{(4+4\sqrt{2}+2)(2+\sqrt{2})}{2\sqrt{2}(4-2)}=\frac{(6+4\sqrt{2})(2+\sqrt{2})}{4\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=$$

$$S_n=\frac{\cancel{2}(3+2\sqrt{2})(2+\sqrt{2})\sqrt{2}}{4\cdot\cancel{2}}=\frac{\sqrt{2}(6+3\sqrt{2}+4\sqrt{2}+4)}{4}=$$

$$S_n=\frac{\sqrt{2}(10+7\sqrt{2})}{4}=\frac{10\sqrt{2}+14}{4}=\frac{\cancel{2}^{1}(5\sqrt{2}+7)}{\cancel{4}_{2}}$$

$$\boxed{S_n=\frac{5\sqrt{2}+7}{2}}$$

2.

1) $1+\sin\frac{\pi}{6}+\sin^2\frac{\pi}{6}+\dots$

$a_1=1$

$q=\frac{\sin\frac{\pi}{6}}{1}=\sin\frac{\pi}{6}$

$S_n=\frac{a_1}{1-q}=\frac{1}{1-\sin\frac{\pi}{6}}=\frac{1}{1-\frac{1}{2}}=\frac{1}{\frac{1}{2}}$

$\boxed{S_n=2}$

2) $1-\cos\frac{\pi}{6}+\cos^2\frac{\pi}{2}-\dots$

$a_1=1$

$q=\frac{-\cos\frac{\pi}{6}}{1}=-\cos\frac{\pi}{6}$

$S_n=\frac{a_1}{1-q}=\frac{1}{1+\cos\frac{\pi}{6}}=\frac{1}{1+\frac{\sqrt{3}}{2}}=\frac{1}{\frac{\sqrt{3}+2}{2}}=$

$\boxed{S_n=\frac{2}{2+\sqrt{3}}}$

3) $1-\sin^2\frac{\pi}{8}+\sin^4\frac{\pi}{8}-\dots$

$a_1=1$

$q=\frac{-\sin^2\frac{\pi}{8}}{1}=-\sin^2\frac{\pi}{8}$

$S_n=\frac{a_1}{1-q}=\frac{1}{1+\sin^2\frac{\pi}{8}}=\Big/\text{iz } \sin^2\frac{x}{2}=\frac{1}{2}(1-\cos x)\Big/=$

$=\frac{1}{1+\frac{1}{2}(1-\cos\frac{\pi}{4})}=\frac{1}{1+\frac{1}{2}(1-\frac{\sqrt{2}}{2})}=$

$=\frac{1}{1+\frac{1}{2}-\frac{\sqrt{2}}{4}}=\frac{1}{\frac{3}{2}-\frac{\sqrt{2}}{4}}=\frac{1}{\frac{6-\sqrt{2}}{4}}=$

$\boxed{S_n=\frac{4}{6-\sqrt{2}}}$