52. zadatak rješen u zbirci

53.

$a, b, 12 \rightarrow$ geom. niz $\Rightarrow b^2 = 12a$

$a, b, 9 \rightarrow$ aritm. niz $\Rightarrow 2b = a + 9 \,/: 2$

$a_{1,2} = \dfrac{30 \pm \sqrt{576}}{2}$

$a_{1,2} = \dfrac{30 \pm 24}{2}$

$a_1 = 27$

$a_2 = 3$

$b_1 = \dfrac{27+9}{2} = 18$

$b_2 = \dfrac{3+9}{2} = 6$

$b = \dfrac{a+9}{2}$

$\left(\dfrac{a+9}{2}\right)^2 = 12a$

$\dfrac{a^2 + 18a + 81}{4} = 12a \,/\cdot$

$a^2 + 18a + 81 - 48a = 0$

$a^2 - 30a + 81 = 0$

$a_{1,2} = \dfrac{30 \pm \sqrt{900 - 324}}{2}$

to su nizovi:

$3, 6, 9$; $27, 18, 9$

$3, 6, 12$; $27, 18, 12$

54.

$a + b + c = 21$ suma aritmetičkog niza

$a + a+d + a+2d = 21 \Rightarrow (S_3)_A = 21$

$a + (a+d)-1 + (a+2d)+1 = 21 \Rightarrow (S_3)_G = 21$

suma geometrijskog niza

$2b = a+c$ (Aritmetički niz)

$(b-1)^2 = a(c+1)$ (geometrijski niz)

$\underbrace{a + c} + b = 21$

$2b + b = 21$

$3b = 21$

$\boxed{b = 7}$

$(b-1)^2 = a(c+1)$

$36 = a(c+1)$

$c+1 = \dfrac{36}{a}$

$c = \dfrac{36}{a} - 1$

$a + 7 + \dfrac{36}{a} - 1 = 21$

$a + 6 + \dfrac{36}{a} - 21 = 0$

$a - 15 + \dfrac{36}{a} = 0 \,/\cdot a$

$a^2 - 15a + 36 = 0$

$a_{1,2} = \dfrac{15 \pm \sqrt{225 - 144}}{2}$

$a_{1,2} = \dfrac{15 \pm \sqrt{81}}{2}$

$a_{1,2} = \dfrac{15 \pm 9}{2}$

$a_1 = 12 \Rightarrow c_1 = 2$

$a_2 = 3 \Rightarrow c_2 = 11$

(geometrijski)

$12, 7, 2 \quad (12, 6, 4)$

$3, 7, 11 \quad (3, 6, 12)$

55.

$$a + b + c = 15 \implies 2b = a + c$$

$$(b-1)^2 = (a-1)(c+1)$$

$$2b + b = 15$$
$$\boxed{b = 5}$$

$$(5-1)^2 = (a-1)(c+1)$$
$$16 = ac + a - c - 1$$
$$17 = ac + a - c$$
$$17 = c(a-1) + a$$
$$17 - a = c(a-1)$$

$$a + 5 + \frac{17-a}{a-1} = 15 \Big/ (a-1)$$

$$\boxed{\frac{17-a}{a-1} = c}$$

$$a^2 - a + 5a - 5 + 17 - a = 15a - 15$$
$$a^2 + 3a + 12 - 15a + 15 = 0$$
$$a^2 - 12a + 27 = 0$$

$$a_{1,2} = \frac{12 \pm \sqrt{144 - 108}}{2} = \frac{12 \pm \sqrt{36}}{2} = \frac{12 \pm 6}{2}$$

$$a_1 = 9 \implies c_1 = \frac{17-9}{9-1} = \frac{8}{8} = 1$$

$$a_2 = 3 \implies c_2 = \frac{17-3}{3-1} = \frac{14}{2} = 7$$

$$\boxed{\begin{array}{c} 9, 5, 1 \\ 3, 5, 7 \end{array}}$$

56.

$$a + b + c = 28 \quad i \ \text{uvjet:} \ b^2 = ac$$

$$4 + 2b + b = 28$$
$$3b = 24$$
$$\boxed{b = 8}$$

$$2b = a + (c-4)$$
$$2b = a + c - 4$$
$$4 + 2b = a + c$$

$$64 = ac$$
$$\boxed{c = \frac{64}{a}}$$

$$a + 8 + \frac{64}{a} = 28 \Big/ a$$
$$a^2 + 8a + 64 - 28a = 0$$
$$a^2 - 20a + 64 = 0$$

$$a_{1,2} = \frac{20 \pm \sqrt{400 - 256}}{2}$$

$$a_{1,2} = \frac{20 \pm 12}{2} \implies \begin{array}{l} a_1 = 16, \ c_1 = 4 \\ a_2 = 4, \ c_2 = 16 \end{array}$$

$$\boxed{\begin{array}{c} 16, 8, 4 \\ 4, 8, 16 \end{array}}$$

57.

$$a = 2 \qquad 2b = a+c \qquad (b-4)^2 = a \cdot c$$
$$2b = 2+c \qquad (b-4)^2 = 2c$$
$$2b-2 = c$$

$$b^2 - 8b + 16 - 2c = 0$$

| 2, 10, 18 Aritmetick. |
| 2, 2, 2 → Můje mi2 |

| 2, 6, 18 Geometrijski |

$$b^2 - 8b + 16 - 4b + 4 = 0$$
$$b^2 - 12b + 20 = 0$$
$$b_{1,2} = \frac{12 \pm \sqrt{144 - 80}}{2}$$
$$b_{1,2} = \frac{12 \pm 8}{2}$$

| $b_1 = 10$ $\quad c_1 = 18$ |
| $b_2 = 2$ $\quad c_2 = 2$ |

58.

$$a + b + c = 42 \quad ; \quad b^2 = ac$$

$$b = a + d$$
$$c = a + 5d$$
$$a + a + d + a + 5d = 42$$
$$3a + 6d = 42 / : 3$$
$$a + 2d = 14$$
$$2d = 14 - a \;/ : 2$$
$$\boxed{d = 7 - \frac{a}{2}}$$

$$a_3 = a_1 + 2d = a + 2d$$
$$2b = a + a + 2d$$
$$2b = 2a + 2d \; / : 2$$
$$b = a + d$$
$$b = 7 - \frac{a}{2} + a$$
$$\boxed{b = 7 + \frac{a}{2}}$$

$$c = a + 5d$$
$$c = a + 5\left(7 - \frac{a}{2}\right)$$
$$c = a + 35 - \frac{5a}{2}$$
$$\boxed{c = 35 - \frac{3a}{2}}$$

mi2
| 2, 8, 32 |

$$\left(7 + \frac{a}{2}\right)^2 = a\left(35 - \frac{3a}{2}\right)$$
$$49 + 7a + \frac{a^2}{4} = 35a - \frac{3a^2}{2} \; / \cdot 4$$
$$196 + 28a + a^2 - 140a + 6a^2 = 0$$
$$7a^2 - 112a + 196 = 0 \; / : 7$$
$$a^2 - 16a + 28 = 0$$
$$a_{1,2} = \frac{16 \pm \sqrt{256 - 112}}{2}$$
$$a_{1,2} = \frac{16 \pm \sqrt{144}}{2}$$
$$a_{1,2} = \frac{16 \pm 12}{2}$$

| $a_1 = 14$ |
| $a_2 = 2$ |

$$b_1 = 7 + 7 \Rightarrow \boxed{b_1 = 14}$$
$$b_2 = 7 + 1 \Rightarrow \boxed{b_2 = 8}$$
$$c_1 = 35 - \frac{3 \cdot 14}{2} = 35 - 21$$
$$\boxed{c_1 = 14}$$
$$c_2 = 35 - \frac{3 \cdot 2}{2} = 35 - 3$$
$$\boxed{c_2 = 32}$$

59.

$$a+b+c=91 \implies b^2=ac$$

$$a+25, \; b+27, \; c+1 \quad \text{aritmetić} \; \text{niz}$$

$$2(b+27)=(a+25)+(c+1)$$

$$2b+54=a+25+c+1$$

$$2b+54=a+c+26$$

$$\underline{2b+28=a+c} \qquad a+c+b=91$$

$$2b+28+b=91$$

$$b^2=ac \qquad\qquad 3b=91-28$$

$$441=ac \qquad\qquad 3b=63$$

$$\boxed{c=\frac{441}{a}} \qquad\qquad \boxed{b=21}$$

$$a+21+\frac{441}{a}=91$$

$$a+\frac{441}{a}-70=0 \; /\cdot a$$

$$a^2-70a+441=0$$

$$a_{1,2}=\frac{70\pm\sqrt{4900-1764}}{2}=\frac{70\pm56}{2}$$

$$\boxed{\begin{array}{c} a_1=63 \\ a_2=7 \end{array}} \quad \boxed{\begin{array}{c} c_1=7 \\ c_2=63 \end{array}}$$

$$\boxed{\begin{array}{c} 63, \; 21, \; 7 \\ \hline 7, \; 21, \; 63 \end{array}} \quad \boxed{\begin{array}{cc} q_1=\frac{21}{63}=\frac{1}{3} & (a_1)_1=63 \\ \hline q_2=\frac{21}{7}=3 & (a_1)_2=7 \end{array}}$$

$$(a_7)_1=a_1 q_1^6=63\cdot\left(\frac{1}{3}\right)^6=63\cdot\frac{1}{3^6}=7\cdot\frac{1}{3^4}=7\cdot\frac{1}{81}=\underline{\underline{\frac{7}{81}}}$$

$$(a_7)_2=a_1 q_2^6=7\cdot 3^6=7\cdot729=\underline{5103}$$

60. i 61. zadatak rješeni su u rješenjima zbirke

62.

a, b, c, d

$b^2 = ac$

$2c = b + d$

$a + d = 14 \Rightarrow d = 14 - a$

$b + c = 12 \Rightarrow c = 12 - b$

$b^2 = a(12 - b)$

$2(12 - b) = b + 14 - a$

$24 - 2b = b + 14 - a$

$-3b = -10 - a$

$\boxed{a = 3b - 10}$

$b^2 = (3b - 10)(12 - b)$

$b^2 = 36b - 3b^2 - 120 + 10b$

$4b^2 - 46b + 120 = 0 \quad /:2$

$2b^2 - 23b + 60 = 0$

$b_{1,2} = \dfrac{23 \pm \sqrt{529 - 480}}{4}$

$b_{1,2} = \dfrac{23 \pm 7}{4}$

$\boxed{b_1 = \dfrac{15}{2}}$

$\boxed{b_2 = 4}$

$a_1 = 3 \cdot \dfrac{15}{2} - 10 = \dfrac{45}{2} - 10$

$\boxed{a_1 = \dfrac{25}{2}}$

$a_2 = 3 \cdot 4 - 10 = 12 - 10$

$\boxed{a_2 = 2}$

$c_1 = 12 - \dfrac{15}{2} = \dfrac{24 - 15}{2}$

$\boxed{c_1 = \dfrac{9}{2}}$

$c_2 = 12 - 4$

$\boxed{c_2 = 8}$

$d_1 = 14 - \dfrac{25}{2}$

$\boxed{d_1 = \dfrac{3}{2}}$

$d_2 = 14 - 2$

$\boxed{d_2 = 12}$

$\boxed{\dfrac{25}{2}, \dfrac{15}{2}, \dfrac{9}{2}, \dfrac{3}{2}}$

$\boxed{2, 4, 8, 12}$

63.

a, b, c glan. niz $\Rightarrow b^2 = ac$ $\quad$ (1)

$a, b+8, c$ arit. niz $\Rightarrow 2(b + 8) = a + c \quad /:2$ $\quad$ (2)

$a, b+8, c+64$ glan. niz $\Rightarrow (b+8)^2 = a(c + 64)$ $\quad$ (3)

$b + 8 = \dfrac{a+c}{2}$ (2) $\Rightarrow b = \dfrac{a+c}{2} - 8$

(1) $\left(\dfrac{a+c}{2} - 8\right)^2 = ac$

$\left(\dfrac{a+c-16}{2}\right)^2 = ac \quad /\cdot 4$

$a^2 + c^2 + 256 + 2ac - 32a - 32c = 4ac$

$a^2 + c^2 - 2ac + 256 - 32a - 32c = 0$

$\left(\dfrac{a+c}{2}\right)^2 = a(c + 64)$ (3)

$\dfrac{(a+c)^2}{4} = a(c + 64) \quad /\cdot 4$

$a^2 + 2ac + c^2 = 4ac + 256a$

$a^2 - 2ac + c^2 = 256a$

$(a - c)^2 = 256a$

$(a-c)^2 + 256 - 32(a+c) = 0$

$256a + 256 - 32a - 32c = 0 \ / 32$

$8a + 8 - a - c = 0$

$\boxed{7a + 8 = c}$

$b = \dfrac{a + 7a + 8}{2} - 8 = \dfrac{8a + 8}{2} - 8 = 4a + 4 - 8$

$\boxed{b = 4a - 4}$

(3) $\quad (4a - 4 + 8)^2 = a(7a + 8 + 64)$

$(4a + 4)^2 = a(7a + 72)$

$16a^2 + 32a + 16 = 7a^2 + 72a$

$9a^2 - 40a + 16 = 0$

$a_{1,2} = \dfrac{40 \pm \sqrt{1600 - 576}}{18} = \dfrac{40 \pm 32}{18}$

$a_1 = \dfrac{72}{18} = 4$

$a_2 = \dfrac{8}{18} = \dfrac{4}{9}$

$b_1 = 4 \cdot 4 - 4 = 12$

$b_2 = 4 \cdot \dfrac{4}{9} - 4 = \dfrac{-20}{9}$

$\boxed{\begin{array}{c} 4, \ 12, \ 36 \\ \hline \dfrac{4}{9}, \ \dfrac{-20}{9}, \ \dfrac{100}{9} \end{array}}$

$c_1 = 7 \cdot 4 + 8 = 36$

$c_2 = 7 \cdot \dfrac{4}{9} + 8 = \dfrac{100}{9}$

64.

a, b, c čine geom. niz $\Rightarrow b^2 = ac$ (1)

$a, b, c-4$ " - arit. " - $\Rightarrow 2b = a + c - 4$ (2)

$a, b-1, c-4-1$ " - geom. " - $\Rightarrow \underline{(b-1)^2 = a(c-5)}$ (3)

iz (2) $\quad b = \dfrac{a+c-4}{2}$

$\left(\dfrac{a+c-4}{2}\right)^2 = ac \ / 4$

$a^2 + c^2 + 16 + 2ac - 8a - 8c = 4ac$

$a^2 + c^2 - 2ac - 8(a+c) + 16 = 0$

$(a-c)^2 - 8(a+c) + 16 = 0$

$(a-c)^2 = 8(a+c) - 16$

$$\left(\frac{a+c-4}{2} - 1\right)^2 = a(c-5)$$

$$\left(\frac{a+c-4-2}{2}\right)^2 = a(c-5)$$

$$\frac{(a+c-6)^2}{4} = a(c-5) \quad /\cdot 4$$

$$a^2 + c^2 + 36 + 2ac - 12a - 12c = 4ac - 20a$$

$$a^2 + c^2 - 2ac - 12(a+c) + 20a + 36 = 0$$

$$(a-c)^2 - 12(a+c) + 20a + 36 = 0$$

$$8(a+c) - 16 - 12(a+c) + 20a + 36 = 0$$

$$-4(a+c) + 20 + 20a = 0 \quad /\cdot (\div 4)$$

$$a + c - 5 - 5a = 0$$

$$\boxed{c = 4a + 5}$$

$$b = \frac{a + 4a + 5 - 4}{2} = \frac{5a+1}{2}$$

$$\boxed{b = \frac{5a+1}{2}}$$

$$\left(\frac{5a+1}{2} - 1\right)^2 = a(4a + \cancel{5} - \cancel{5})$$

$$\left(\frac{5a+1-2}{2}\right)^2 = 4a^2 \quad /\cdot 4$$

$$25a^2 - 10a + 1 = 16a^2$$

$$9a^2 - 10a + 1 = 0$$

$$a_{1/2} = \frac{10 \pm \sqrt{100-36}}{18} = \frac{10 \pm 8}{18}$$

$$a_1 = 1$$
$$a_2 = \frac{1}{9}$$

$$c_1 = 9$$
$$c_2 = \frac{49}{9}$$

$$b_1 = 3$$
$$b_2 = \frac{7}{9}$$

$$\boxed{\begin{array}{ccc} 1, & 3, & 9 \\ \frac{1}{9}, & \frac{7}{9}, & \frac{49}{9} \end{array}}$$

66.

$a = a_1 = 5$ $a_1 = a = 5$

$b = a_1 + 3d = 5 + 3d$ $b = a_1 q^2 = 5q^2$

$c = a_1 + 15d = 5 + 15d$ $c = a_1 q^4 = 5q^4$

$5 + 3d = 5q^2 \Rightarrow q^2 = \dfrac{5 + 3d}{5}$

$5 + 15d = 5q^4$

$5 + 15d = 5 \cdot \left(\dfrac{5 + 3d}{5}\right)^2$

$5 + 15d = 5 \cdot \dfrac{25 + 30d + 9d^2}{25} \quad / \cdot 5$

$25 + 75d - 25 - 30d - 9d^2 = 0$

$-9d^2 + 45d = 0$

$-9d(d - 5) = 0$

$d_1 = 0 \Rightarrow$ nije rješenje

$\boxed{d_2 = 5}$

$q^2 = \dfrac{5 + 3 \cdot 5}{5} = \dfrac{20}{5} = 4$

$\boxed{\dfrac{q_1 = -2}{q_2 = 2}}$

$b = 5 + 3 \cdot 5 = 20$

$c = 5 + 15 \cdot 5 = 80$

$\boxed{5, 20, 80}$

67.

$a_2 = 8 = a_1 q$

$a_5 = 512 = a_1 q^4$ $\Rightarrow q^3 = \dfrac{512}{8} = 64$

$\boxed{q = 4} \Rightarrow \boxed{d = 2}$

$a_2 = 8 = a_1 \cdot 4$

$\boxed{a_1 = 2}$ $a_3 = a_1 q^2 = 2 \cdot 4^2 = 32$

$a_1 + a_2 + a_3 = 2 + 8 + 32 = \underline{\underline{42}}$

$S_3 = \dfrac{n}{2}(2a_1 + (n-1)d)$

$S_3 = \dfrac{3}{2}(2a_1 + (3-1) \cdot 2) = 42 \quad / \cdot 2$

$3(2a_1 + 4) = 84 \quad / : 3$

$2a_1 + 4 = 28$

$2a_1 = 24$

$\boxed{a_1 = 12}$

$a_2 = a_1 + d = 12 + 2 = 14$

$a_3 = a_2 + d = 14 + 2 = 16$

$\boxed{12, 14, 16}$

68.

$a_1 = a = 24$

$a_5 = a_1 + 4d = 24 + 4d = 24q$

$a_{11} = a_1 + 10d = 24 + 10d = 24q^2$

$(24q)^2 = 24 \cdot 24q^2$

$(24 + 4d)^2 = 24 \cdot (24 + 10d)$

$576 + 192d + 16d^2 = 576 + 240d$

$16d^2 - 48d = 0$

$16d(d - 3) = 0$

$d_1 = 0$

$\boxed{d_2 = 3}$

$\underline{24}, 27, 30, 33, \underline{36}, 39, 42, 45, 48, 51, \underline{54}$

69.

$(S_9)_1 = 369$

$a_1 = 1$

$a_9 = \frac{9}{2}(a_1 + a_n) = 369$

$\frac{9}{2}(1 + a_n) = 369 / \cdot 2$

$9 + 9a_n = 738$

$9a_n = 729 / : 9$

$\boxed{a_n = 81}$

$\boxed{a_9 = 81}$

$a_9 = a_1 q^8 = 81$

$1 \cdot q^8 = 81$

$q^8 = 81$

$q = \sqrt[8]{3^4}$

$\boxed{q = \sqrt{3}}$