2. POTENCIJE I ALGEBARSKI IZRAZI

2.1. Potencije

1.

1.) $\underbrace{a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a}_{\text{prebrojimo faktore koji se mnoze ima ih sedam}} = a^7$ **Zadatak rjesavamo po pravilu opisanom ispod ovog zadatka.**

Pravilo glasi: Uzmi faktor *a* koji se ponavlja kao baza, izbroji koliko se puta javlja taj jednaki faktor (7 puta) i stavi rezultat tog prebrojavanja kao eksponent.

2.) $(ab)\cdot(ab)\cdot(ab)\cdot(ab)\cdot(ab) = (ab)^5$

3.) $(x-y)\cdot(x-y)\cdot(x-y)\cdot(x-y) = (x-y)^4$

4.) $(n^2+1)\cdot(n^2+1)\cdot(n^2+1)\cdot(n^2+1)\cdot(n^2+1) = (n^2+1)$

5.) $\frac{a}{a+2}\cdot\frac{a}{a+2}\cdot\frac{a}{a+2} = \left(\frac{a}{a+2}\right)^3$

2.

$x^1 \rightarrow$ osnovica je x , eksponent je 11

$(abc)^4 \rightarrow$ osnovica je (abc) , eksponent je 4

$(a^3+b^3)^5 \rightarrow$ osnovica je (a^3+b^3), eksponent je 5

$(x^n-y^n)^4 \rightarrow$ osnovica je (x^n-y^n) , eksponent je 4

12.

1.) $2^{11}\cdot 5^{11} = (2\cdot 5)^{11} = 10^{11} \Rightarrow$ broj ima $11+1 = 12$ znamenki

Pogledajmo primjere : $10^2 = 100 \rightarrow$ broj ima tri znamenke

$10^3 = 1000 \rightarrow$ broj ima četri znamenke

Zaključak: broj 10^n – ima $n+1$ znamenki

2.) $2^{25}\cdot 5^{20} = 2^{5+20}\cdot 5^{20} = 2^5\cdot 2^{20}\cdot 5^{20} = 2^5\cdot(2\cdot 5)^{20} = 32\cdot 10^{20} \Rightarrow$ ovaj broj je jednak

32 00…0 tj. 32 i 20 nula $\Rightarrow$ dakle droj znamenki $= 22$

3.) $2^{10}\cdot 5^{10}\cdot 10^{15} = (2\cdot 5)^{10}\cdot 10^{15} = 10^{10}\cdot 10^{15} = 10^{10+15} = 10^{25} \Rightarrow$ broj ima $25+1 = 26$ znamenki

4) $2^5\cdot 5^5 = (2\cdot 5)^5 = 10^5$

$\Rightarrow$ znamo (pogledaj zadatak 15. 1)) broj 10^n ima $n+1$ znamenki.

$\Rightarrow$ znači 10^5 ima $n=5$, tj 6 znamenki

5) $4^7\cdot 5^{10} = (2\cdot 2)^7\cdot(5)^{10} = (2^2)^7\cdot(5^5)^2 = (2^7)^2\cdot(5^5)^2$

$=(128)^2\cdot(3125)^2 = (400\,000)^2 =$

$=(4\cdot 10^5)^2 = 4^2\cdot(10^5)^2 = 16\cdot 10^{10}$

$\Rightarrow$ broj $X\cdot 10^n$, ako je X dvoznamenkast broj ima $2+(n+1)$ znamenki, znači:

$2+10+1 = 13$

broj: $4^7\cdot 5^{10}$ ima 13 znamenki

6) $2^{12}\cdot 25^8 = 2^{12}\cdot(5^2)^8 = (2^6)^2\cdot(5^8)^2 =$

$=(64)^2\cdot(390625)^2 =$

$=(64\cdot 390\,625)^2 =$

$=(25000000)^2 = (25\cdot 10^6)^2 =$

$=25^2\cdot(10^6)^2 = 625\cdot 10^{12}$

$\Rightarrow$ broj $X\cdot 10^n$ ako je X troznamenkast broj ima $3+(n+1)$ znamenki; znači:

$3+12+1 = 16.$

$\Rightarrow$ broj $2^{12}\cdot 25^8$ ima 16 znamenki

14. Primjenimo pravilo: $\left(a^n\right)^m = a^{n\cdot m}$

$\left(2^3\right)^4 = 2^{3\cdot 4} = 2^{12}$

1) $\left(3^4\right)^3 = 3^{4\cdot 3} = 3^{12}$

2) $\left(8^2\right)^2 = 8^{2\cdot 2} = 8^4 = \left(2^3\right)^4 = 2^{3\cdot 4} = 2^{12}$

3) $\left(10^3\right)^4 = 10^{3\cdot 4} = 10^{12}$

$\left(5^5\right)^2 = 5^{5\cdot 2} = 5^{10}$

4) $\left(a^{n+1}\right)^3 = a^{(n+1)\cdot 3} = a^{3n+3}$

5) $\left(a^4\right)^{n+1} = a^{4\cdot(n+1)} = a^{4n+4}$

6) $\left(a^{n-1}\right)^{n+1} = a^{(n-1)\cdot(n+1)} = a^{n^2-1}$

15. U ovom zadatku koristimo dva pravila: $\left(a^n\right)^m = a^{n\cdot m}$ i $a^n \cdot a^m = a^{n+m}$

1.) $\left(3^3\right)^4 \cdot \left(3^4\right)^3 = 3^{3\cdot 4} \cdot 3^{4\cdot 3} = 3^{12} \cdot 3^{12} = 3^{12+12} = 3^{24}$

2.) $\left(2^5\right)^3 \cdot \left(2^3\right)^3 = 2^{5\cdot 3} \cdot 2^{3\cdot 3} = 2^{15} \cdot 2^9 = 2^{15+9} = 2^{24}$

$\left(5^2\right)^2 \cdot \left(5^5\right)^3 = 5^{2\cdot 2} \cdot 5^{5\cdot 3} = 5^4 \cdot 5^{15} = 5^{4+15} = 5^{19}$

3) $\left(10^{n+2}\right)^3 \cdot \left(10^2\right)^{n-1} = 10^{(n+2)\cdot 3} \cdot 10^{2\cdot(n-1)} = 10^{3n+6} \cdot 10^{2n-2} = 10^{3n+6+2n-2} = 10^{3n+2n+6-2} = 10^{5n+4}$

4) $\left(4^{n-1}\right)^2 \cdot \left(4^2\right)^{n+1} = 4^{(n-1)\cdot 2} \cdot 4^{2\cdot(n+1)} = 4^{2n-2} \cdot 4^{2n+2} = 4^{2n-2+2n+2} = 4^{4n} = \left(2^2\right)^{4n} = 2^{2\cdot 2n} = 2^{8n}$

16.

1) $\left(16\cdot4^3\cdot8^2\right)^5=\left(2^4\cdot\left(2^2\right)^3\cdot\left(2^3\right)^2\right)^5=\left(2^4\cdot2^{2\cdot3}\cdot2^{3\cdot2}\right)^5=\left(2^4\cdot2^6\cdot2^6\right)^5=\left(2^{4+6+6}\right)^5=$

$=\left(2^{16}\right)^5=2^{16\cdot5}=2^{80}$

$\left(8^2\cdot2\cdot4^3\right)^2=\left(\left(2^3\right)^2\cdot2^1\cdot\left(2^2\right)^3\right)^2=\left(2^{3\cdot2}\cdot2^1\cdot2^{2\cdot3}\right)^2=\left(2^6\cdot2^1\cdot2^6\right)^2=\left(2^{6+1+6}\right)^2=$

$=\left(2^{13}\right)^2=2^{13\cdot2}=2^{26}$

2) $\left(16^2\cdot4^3\cdot8^4\right)^3=\left(\left(2^4\right)^2\cdot\left(2^2\right)^3\cdot\left(2^3\right)^4\right)^3=\left(2^{4\cdot2}\cdot2^{2\cdot3}\cdot2^{3\cdot4}\right)^3=\left(2^8\cdot2^6\cdot2^{12}\right)^3=$

$=\left(2^{8+6+12}\right)^3=\left(2^{26}\right)^3=2^{26\cdot3}=2^{78}$

17.

1.) $\left(27^2\cdot81\cdot9^3\right)^4=\left(\left(3^3\right)^2\cdot3^4\cdot\left(3^2\right)^3\right)^4=\left(3^{3\cdot2}\cdot3^4\cdot3^{2\cdot3}\right)^4=\left(3^6\cdot3^4\cdot3^6\right)^4=\left(3^{6+4+6}\right)^4=\left(3^{16}\right)^4=$

$=3^{16\cdot4}=3^{64}$

2.) $\left(9^3\cdot3\cdot27^2\right)^3=\left(\left(3^2\right)^3\cdot3^1\cdot\left(3^3\right)^2\right)^3=\left(3^{2\cdot3}\cdot3^1\cdot3^{3\cdot2}\right)^3=\left(3^6\cdot3^1\cdot3^6\right)^3=\left(3^{6+1+6}\right)^3=$

$=\left(3^{13}\right)^3=3^{13\cdot3}=3^{39}$

3.) $\left(3^5\cdot9^5\cdot27^5\right)^2=\left(3^5\cdot\left(3^2\right)^5\cdot\left(3^3\right)^5\right)^2=\left(3^5\cdot3^{2\cdot5}\cdot3^{3\cdot5}\right)^2=\left(3^5\cdot3^{10}\cdot3^{15}\right)^2=\left(3^{5+10+15}\right)^2=$

$=\left(3^{30}\right)^2=3^{30\cdot2}=3^{60}$

8.) $\left(-4^4\right)^3+\left(-2^3\right)^8+\left(-8\right)^8+2^{24}=\left(-1\cdot\left(2^2\right)^4\right)^3+\left(-1\cdot2^3\right)^8+\left(-1\cdot2^3\right)^8+2^{24}=$

$=\left(-1\right)^3\cdot\left(\left(2^2\right)^4\right)^3+\left(-1\right)^8\cdot\left(2^3\right)^8+\left(-1\right)^8\cdot\left(2^3\right)^8+2^{24}=$

$=-1\cdot2^{2\cdot4\cdot3}+1\cdot2^{3\cdot8}+1\cdot2^{3\cdot8}+2^{24}=$

$=-1\cdot2^{24}+1\cdot2^{24}+1\cdot2^{24}+1\cdot2^{24}=$

$=\left(-1+1+1+1\right)\cdot2^{24}=2\cdot2^{24}=2^1\cdot2^{24}=2^{1+24}=2^{25}$

27.

1.) $\left(-a^2\right)^{2n+1}+\left(-a\right)^{4n}\cdot\left(-a\right)^2=\left(-1\cdot a^2\right)^{2n+1}+\left(-a\right)^{4n+2}=$

$=\left(-1\right)^{2n+1}\cdot\left(a^2\right)^{2n+1}+\left(-1\cdot a\right)^{4n+2}=$ $\left\{\begin{array}{l}\text{eksponent } 2n+1 \text{ je neparan broj}\\ \text{pa je } \left(-1\right)^{2n+1}=-1\end{array}\right\}$

$=-1\cdot a^{2\cdot(2n+1)}+\left(-1\right)^{4n+2}\cdot a^{4n+2}=$

$=-1\cdot a^{4n+2}+1\cdot a^{4n+2}=\left(-1+1\right)\cdot a^{4n+2}=0\cdot a^{4n+2}=0$

2.) $\left(-a^3\right)^{2n+2}\cdot\left(-a\right)^3-\left(-a^{2n+3}\right)^3=\left(-1\cdot a^3\right)^{2n+2}\cdot\left(-1\cdot a\right)^3-\left(-1\cdot a^{2n+3}\right)^3=$

$=\left(-1\right)^{2n+2}\cdot\left(a^3\right)^{2n+2}\cdot\left(-1\right)^3\cdot a^3-\left(-1\right)^3\cdot\left(a^{2n+3}\right)^3=$

$=1\cdot a^{3\cdot(2n+2)}\cdot\left(-1\right)\cdot a^3-\left(-1\right)\cdot a^{(2n+3)\cdot3}=$

$=1\cdot\left(-1\right)\cdot3^{6n+6}\cdot a^3+1\cdot a^{6n+9}=-1\cdot3^{6n+6+3}+1\cdot a^{6n+9}=$

$=-1\cdot3^{6n+9}+1\cdot3^{6n+9}=\left(-1+1\right)\cdot3^{6n+9}=0\cdot3^{6n+9}=0$

3.) $\left(-a\right)^{2n+2}\cdot\left(-a^{2n-1}\right)^2-\left(-a^{n+1}\right)^3\cdot\left(a^3\right)^{n-1}=\left(-1\cdot a\right)^{2n+2}\cdot\left(-1\cdot a^{2n-1}\right)^2-\left(-1\cdot a^{n+1}\right)^3\cdot a^{3\cdot(n-1)}=$

$=\left(-1\right)^{2n+2}\cdot a^{2n+2}\cdot\left(-1\right)^2\cdot\left(a^{2n-1}\right)^2-\left(-1\right)^3\cdot\left(a^{n+1}\right)^3\cdot a^{3n-3}=$

$=1\cdot a^{2n+2}\cdot1\cdot\left(a^{2n-1}\right)^2-\left(-1\right)\cdot a^{(n+1)\cdot3}\cdot a^{3n-3}=$

$=a^{2n+2}\cdot a^{(2n-1)\cdot2}+1\cdot a^{3n+3}\cdot a^{3n-3}=$

$=a^{2n+2}\cdot a^{4n-2}+a^{3n+3+3n-3}=$

$=a^{2n+2+4n-2}+a^{6n}=a^{6n}+a^{6n}=2\cdot a^{6n}$

4.) $-\left(a^{n-1}\right)^2\cdot\left(a^n\right)^2\cdot\left(-a^{n-1}\right)^3-\left(-a^{n-1}\right)^3\cdot\left(-a^{2n-1}\right)^2=$

$=-\left(a^{(n-1)\cdot 2}\right)\cdot a^{2n}\cdot\left(-1\cdot a^{n-1}\right)^3-\left(-1\cdot a^{n-1}\right)^3\cdot\left(-1\cdot a^{2n-1}\right)^2=$

$=-1\cdot a^{2n-2}\cdot a^{2n}\cdot(-1)^3\cdot\left(a^{n-1}\right)^3-(-1)^3\cdot\left(a^{n-1}\right)^3\cdot(-1)^2\cdot\left(a^{2n-1}\right)^2=$

$=-1\cdot a^{2n-2+2n}\cdot(-1)\cdot a^{(n-1)\cdot 3}-(-1)\cdot a^{(n-1)\cdot 3}\cdot 1\cdot a^{(2n-1)\cdot 2}=$

$=-1\cdot(-1)\cdot a^{4n-2}\cdot a^{3n-3}+1\cdot a^{3n-3}\cdot a^{4n-2}=$

$=1\cdot a^{4n-2+3n-3}+1\cdot a^{3n-3+4n-2}=$

$=1\cdot a^{7n-5}+1\cdot a^{7n-5}=(1+1)\cdot a^{7n-5}=2\cdot a^{7n-5}$

28.

1.) $3\cdot 2^6+10\cdot 2^5=3\cdot 2^{1+5}+10\cdot 2^5=3\cdot 2^1\cdot 2^5+10\cdot 2^5=6\cdot 2^5+10\cdot 2^5=(6+10)\cdot 2^5=16\cdot 2^5=$

$=2^4\cdot 2^5=2^{4+5}=2^9$

2.) $11\cdot 4^6+20\cdot 2^{10}=11\cdot\left(2^2\right)^6+5\cdot 4\cdot 2^{10}=11\cdot 2^{2\cdot 6}+5\cdot 2^2\cdot 2^{10}=11\cdot 2^{12}+5\cdot 2^{2+10}=11\cdot 2^{12}+5\cdot 2^{12}=$

$=(11+5)\cdot 2^{12}=16\cdot 2^{12}=2^4\cdot 2^{12}=2^{4+12}=2^{16}$

3.) $6\cdot 2^{11}+5\cdot 4^6=3\cdot 2\cdot 2^{11}+5\cdot\left(2^2\right)^6=3\cdot 2^1\cdot 2^{11}+5\cdot 2^{2\cdot 6}=3\cdot 2^{1+11}+5\cdot 2^{12}=3\cdot 2^{12}+5\cdot 2^{12}=$

$=(3+5)\cdot 2^{12}=8\cdot 2^{12}=2^3\cdot 2^{12}=2^{3+12}=2^{15}$

4.) $2^{13}+4\cdot 2^{11}=2^{13}+2^2\cdot 2^{11}=2^{13}+2^2\cdot 2^{11}=2^{13}+2^{2+11}=$

$=2^{13}+2^{13}=2\cdot 2^{13}=2^1\cdot 2^{13}=2^{1+13}=2^{14}$

31.

1.) $3^{n-1}\cdot 2^{n+1}-3^{n+1}\cdot 2^{n-1}-6^{n-1}=3^{n-1}\cdot 2^{n-1+2}-3^{n-1+2}\cdot 2^{n-1}-6^{n-1}=$

$=3^{n-1}\cdot 2^{n-1}\cdot 2^{2}-3^{n-1}\cdot 3^{2}\cdot 2^{n-1}-6^{n-1}=$

$=(3\cdot 2)^{n-1}\cdot 2^{2}-3^{2}\cdot(3\cdot 2)^{n-1}-6^{n-1}=$

$=6^{n-1}\cdot 4-9\cdot 6^{n-1}-1\cdot 6^{n-1}=$

$=(4-9-1)\cdot 6^{n-1}=-6\cdot 6^{n-1}=-1\cdot 6^{1}\cdot 6^{n-1}=$

$=-1\cdot 6^{1+n-1}=-1\cdot 6^{n+1-1}=-1\cdot 6^{n}=-6^{n}$

2.) $2^{2n-1}\cdot 9^{n-1}+4^{n}\cdot 3^{2n-2}=2^{2n}\cdot 2^{-1}\cdot 9^{n}\cdot 9^{-1}+(2^{2})^{n}\cdot 3^{2n}\cdot 3^{-2}=2^{2n}\cdot\frac{1}{2}\cdot(3^{2})^{n}\cdot\frac{1}{9}+2^{2\cdot n}\cdot 3^{2n}\cdot\frac{1}{3^{2}}=$

$=\frac{1}{2}\cdot\frac{1}{9}\cdot 2^{2n}\cdot 3^{2n}+2^{2n}\cdot 3^{2n}\cdot\frac{1}{9}=\frac{1}{18}\cdot(2\cdot 3)^{2n}+\frac{1}{9}\cdot(2\cdot 3)^{2n}=$

$=\frac{1}{18}\cdot 6^{2n}+\frac{1}{9}\cdot 6^{2n}=\left(\frac{1}{18}+\frac{1}{9}\right)\cdot 6^{2n}=$

$=\frac{3}{18}\cdot 6^{2n}=\frac{1}{6}\cdot 6^{2n}=6^{-1}\cdot 6^{2n}=6^{-1+2n}=6^{2n-1}$

3.) $4^{n}\cdot 9^{n-1}+4^{n+1}\cdot 9^{n-1}+16\cdot 36^{n-1}=4^{n}\cdot 9^{n}\cdot 9^{-1}+4^{n}\cdot 4^{1}\cdot 9^{n}\cdot 9^{-1}+16\cdot 36^{n}\cdot 36^{-1}=$

$=(4\cdot 9)^{n}\cdot\frac{1}{9}+4\cdot\frac{1}{9}\cdot(4\cdot 9)^{n}+16\cdot\frac{1}{36}\cdot 36^{n}=$

$=\frac{1}{9}\cdot 36^{n}+\frac{4}{9}\cdot 36^{n}+\frac{4}{9}\cdot 36^{n}=$

$=\left(\frac{1}{9}+\frac{4}{9}+\frac{4}{9}\right)\cdot 36^{n}=\frac{9}{9}\cdot 36^{n}=1\cdot 36^{n}=36^{n}=(6^{2})^{n}=6^{2n}$

32.

1.) $2^{n-1}\cdot 5^{n+2}-5^{n+1}\cdot 2^{n-1}=2^{n}\cdot 2^{-1}\cdot 5^{n}\cdot 5^{2}-5^{n}\cdot 5^{1}\cdot 2^{n}\cdot 2^{-1}=$

$=\frac{1}{2}\cdot 25\cdot(2\cdot 5)^{n}-5\cdot\frac{1}{2}\cdot(5\cdot 2)^{n}=\frac{25}{2}\cdot 10^{n}-\frac{5}{2}\cdot 10^{n}=$

$=\left(\frac{25}{2}-\frac{5}{2}\right)\cdot 10^{n}=\frac{20}{2}\cdot 10^{n}=10\cdot 10^{n}=10^{1}\cdot 10^{n}=10^{1+n}=10^{n+1}$

36.

1) $\left(\frac{4}{5}x^5y^3\right):\left(\frac{8}{15}x^3y^2\right) = \frac{4}{5}:\frac{8}{15}\cdot x^5:x^3\cdot y^3:y^2 = \frac{4}{5}\cdot\frac{15}{8}\cdot x^{5-3}\cdot y^{3-2} = \frac{3}{2}\cdot x^2\cdot y^1 = \frac{3}{2}x^2y$

2.) $\left(-3x^4y^4\right):\left(\frac{6}{11}xy^2\right) = -3:\frac{6}{11}\cdot x^4:x^1\cdot y^4:y^2 = -3\cdot\frac{11}{6}\cdot x^{4-1}\cdot y^{4-2} = -\frac{11}{2}\cdot x^3\cdot y^2 = -\frac{11}{2}x^3y^2$

3.) $\left(8a^8b^8\right):\left(16a^5b^5\right) = 8:16\cdot a^8:a^5\cdot b^8:b^5 = \frac{8}{16}\cdot a^{8-5}\cdot b^{8-5} = \frac{1}{2}\cdot a^3\cdot b^3 = \frac{1}{2}a^3b^3$

4.) $\left(\frac{9}{16}a^6b^4\right):\left(18a^3b\right) = \frac{9}{16}:18\cdot a^6:a^3\cdot b^4:b^1 = \frac{9}{16}\cdot\frac{1}{18}\cdot a^{6-3}\cdot b^{4-1} = \frac{1}{32}\cdot a^3\cdot b^3 = \frac{1}{32}a^3b^3$

5.) $\left(\frac{5}{24}a^3b^8\right)\cdot\left(-\frac{25}{12}a^2b^5\right) = \frac{5}{24}:\left(-\frac{25}{12}\right)\cdot a^3:a^2\cdot b^8:b^5 = \frac{5}{24}\cdot\left(-\frac{12}{25}\right)\cdot a^{3-2}\cdot b^{8-5} = -\frac{1}{10}\cdot a^1\cdot b^3$

37.

1.) $\frac{27\cdot 3^{2n-1}}{9^{n+1}} = \frac{3^3\cdot 3^{2n-1}}{\left(3^2\right)^{n+1}} = \frac{3^{3+2n-1}}{3^{2\cdot(n+1)}} = \frac{3^{2n+2}}{3^{2n+2}} = 1 \Rightarrow$ (nakon sređivanja izraza dobili smo da je on jednak jedan kako krajnji rezultat u sebi nema (n) to izraz ne ovisi o (n))

2.) $\frac{32\cdot 4^{n-1}}{2^{2n+1}} = \frac{2^5\cdot\left(2^2\right)^{n-1}}{2^{2n+1}} = \frac{2^5\cdot 2^{2n-2}}{2^{2n+1}} = \frac{2^{5+2n-2}}{2^{2n+1}} = \frac{2^{2n+3}}{2^{2n+1}} = \frac{2^{2n+1+2}}{2^{2n+1}} = \frac{2^{2n+1}\cdot 2^2}{2^{2n+1}} = 2^2 = 4$

kratimo ↵

3.) $\frac{25^{3n-1}}{100\cdot 125^{2n-2}} = \frac{\left(5^2\right)^{3n-1}}{4\cdot 25\cdot\left(5^3\right)^{2n-2}} = \frac{5^{2\cdot(3n-1)}}{4\cdot 5^2\cdot 5^{3\cdot(2n-2)}} = \frac{5^{6n-2}}{4\cdot 5^2\cdot 5^{6n-6}} = \frac{5^{6n-2}}{4\cdot 5^{2+6n-6}} = =$

$= \frac{5^{6n-2}}{4\cdot 5^{6n-4}} = \frac{5^{6n}\cdot 5^{-2}}{4\cdot 5^{6n}\cdot 5^{-4}} = \frac{5^{-2}}{4\cdot 5^{-4}} = \frac{\frac{1}{5^2}}{4\cdot\frac{1}{5^4}} = \frac{5^4}{4\cdot 5^2} = \frac{5^{2+2}}{4\cdot 5^2} = \frac{5^2\cdot 5^2}{4\cdot 5^2} = \frac{5^2}{4} = \frac{25}{4}$

kratimo ↵

4)

$\frac{27^{n+3}\cdot 9^{n+4}}{3^{n+1}\cdot 81^{n+2}} = \frac{3^{3n+9}\cdot 3^{2n+8}}{3^{n+1}\cdot 3^{4n+8}} = \frac{3^{5n+17}}{3^{5n+9}} = 3^{5n+17-5n-9} = 3^8$

38.

1.)
$$\frac{2\cdot5^{n+2}+3\cdot5^{n+1}}{3\cdot5^{n-1}-2\cdot5^{n-2}}=\frac{2\cdot5^n\cdot5^2+3\cdot5^n\cdot5^1}{3\cdot5^n\cdot5^{-1}-2\cdot5^n\cdot5^{-2}}=\frac{2\cdot25\cdot5^n+3\cdot5\cdot5^n}{3\cdot\frac{1}{5}\cdot5^n-2\cdot\frac{1}{5^2}\cdot5^n}=\frac{50\cdot5^n+15\cdot5^n}{\frac{3}{5}\cdot5^n-\frac{2}{25}\cdot5^n}=\frac{(50+15)\cdot5^n}{\left(\frac{3}{5}-\frac{2}{25}\right)\cdot5^n}=$$

kratimo
$$=\frac{65\cdot5^n}{\frac{13}{25}\cdot5^n}=\frac{65}{\frac{13}{25}}=\frac{\frac{65}{1}}{\frac{13}{25}}=\frac{25\cdot65}{1\cdot13}=\frac{25\cdot5\cdot13}{13}=25\cdot5=125$$

2.)
$$\frac{(8^n+8^{n-1})^4}{(16^{n-1}-16^{n-2})^3}=\frac{(8^n+8^n\cdot8^{-1})^4}{(16^n\cdot16^{-1}-16^n\cdot16^{-2})^3}=\frac{((1+8^{-1})\cdot8^n)^4}{((16^{-1}-16^{-2})\cdot16^n)^3}=\frac{\left(\left(1+\frac{1}{8}\right)\cdot8^n\right)^4}{\left(\left(\frac{1}{16}-\frac{1}{256}\right)\cdot16^n\right)^3}=$$

$$=\frac{\left(\frac{9}{8}\cdot8^n\right)^4}{\left(\frac{15}{256}\right)^3\cdot(16^n)^3}=\frac{\left(\frac{9}{8}\right)^4\cdot(8^n)^4}{\frac{15^3}{256^3}\cdot((2^4)^n)^3}=\frac{\frac{9^4}{8^4}\cdot((2^3)^n)^4}{\frac{(3\cdot5)^3}{(2^8)^3}\cdot2^{4\cdot n\cdot3}}=\frac{\frac{(3^2)^4}{(2^3)^4}\cdot2^{3\cdot n\cdot4}}{\frac{(3\cdot5)^3}{(2^8)^3}\cdot2^{4\cdot n\cdot3}}=\frac{\frac{3^8}{2^{12}}\cdot2^{12n}}{\frac{3^3\cdot5^3}{2^{8\cdot3}}\cdot2^{12}}=$$

kratimo

$$=\frac{\frac{3^8}{2^{12}}}{\frac{3^3\cdot5^3}{2^{24}}}=\frac{2^{24}\cdot3^8}{2^{12}\cdot3^3\cdot5^3}=\frac{2^{12+12}\cdot3^{5+3}}{2^{12}\cdot3^3\cdot5^3}=\frac{2^{12}\cdot2^{12}\cdot3^5\cdot3^3}{2^{12}\cdot3^3\cdot5^3}=\frac{2^{12}\cdot3^5}{5^3}$$

39.

1.)
$$\frac{5^{2n-1}-25^{n-1}}{125^{n-1}-5^{3n-2}}=\frac{5^{2n}\cdot5^{-1}-25^n\cdot25^{-1}}{125^n\cdot125^{-1}-5^{3n}\cdot5^{-2}}=\frac{5^{2n}\cdot\frac{1}{5}-(5^2)^n\cdot\frac{1}{25}}{(5^3)^n\cdot\frac{1}{125}-5^{3n}\cdot\frac{1}{5^2}}=\frac{\frac{1}{5}\cdot5^{2n}-\frac{1}{25}\cdot5^{2n}}{\frac{1}{125}\cdot5^{3n}-\frac{1}{25}\cdot5^{3n}}=$$

$$=\frac{\left(\frac{1}{5}-\frac{1}{25}\right)\cdot5^{2n}}{\left(\frac{1}{125}-\frac{1}{25}\right)\cdot5^{3n}}=\frac{\frac{5-1}{25}\cdot5^{2n}}{\frac{1-5}{125}\cdot5^{3n}}=\frac{\frac{4}{25}\cdot5^{2n}}{-\frac{4}{125}\cdot5^{3n}}=-\frac{\frac{4\cdot5^{2n}}{25}}{\frac{4\cdot5^{3n}}{125}}=-\frac{125\cdot4\cdot5^{2n}}{25\cdot4\cdot5^{3n}}=$$

$$=-\frac{5\cdot25\cdot4\cdot5^{2n}}{25\cdot4\cdot5^{2n+n}}=-\frac{5\cdot5^{2n}}{5^{2n}\cdot5^n}=-\frac{5}{5^n}=-5^1:(5^n)^{-1}=-5^1\cdot5^{-n}=-5^{1-n}$$

2.2.– *poglavlje*

27.

1) $\left(\frac{1}{3}c^2-\frac{1}{2}d^2\right)^3=\left(\frac{1}{3}c^2\right)^3-3\cdot\left(\frac{1}{3}c^2\right)^2\cdot\frac{1}{2}d^2+3\cdot\frac{1}{3}c^2\cdot\left(\frac{1}{2}d^2\right)^2-\left(\frac{1}{2}d^2\right)^3=$

$=\frac{1^3}{3^3}(c^2)^3-3\cdot\frac{1}{9}c^4\cdot\frac{1}{2}d^2+c^2\cdot\frac{1}{4}d^4-\frac{1^3}{2^3}(d^2)^3=$

$=\frac{1}{27}c^6-\frac{1}{6}c^4d^2+\frac{1}{4}c^2d^4-\frac{1}{8}d^6$

2) $\left(3a^2b-4c^3\right)^3=\left(3a^2b\right)^3-3\cdot\left(3a^2b\right)^2\cdot 4c^3+3\cdot 3a^2b\cdot(4c)^2-\left(4c^3\right)^3=$

$=3^3(a^2)^3b^3-12c^3\cdot 9(a^2)^2b^2+9a^2b\cdot 16c^2-4^3(c^3)^3=$

$=27a^6b^3-108a^4b^2c^3+144a^2bc^2-64c^9$

3) $\left(\frac{2}{3}a^2b^2-\frac{3}{2}c^4\right)^3=$

$=\left(\frac{2}{3}a^2b^2\right)^3-3\cdot\left(\frac{2}{3}a^2b^2\right)^2\cdot\frac{3}{2}c^4+3\cdot\frac{2}{3}a^2b^2\cdot\left(\frac{3}{2}c^4\right)^2-\left(\frac{3}{2}c^4\right)^3=$

$=\frac{2^3}{3^3}(a^2)^3(b^2)^3-3\cdot\frac{4}{9}a^4b^4\cdot\frac{3}{2}c^4+2a^2b^2\cdot\frac{9}{4}c^8-\frac{3^3}{2^3}(c^4)^3=$

$=\frac{8}{27}a^6b^6-2a^4b^4c^4+\frac{9}{2}a^2b^2c^8-\frac{27}{8}c^{12}$

4) $\left(2^m-3^n\right)^3=\left(2^n\right)^3-3\cdot\underset{①}{\left(2^n\right)^2\cdot 3^m}+3\cdot 2^n\cdot\left(3^n\right)^2-\underset{②}{\left(3^n\right)^3}=$

$=\left(2^3\right)^n-3\cdot\left(2^2\right)^n\cdot 3^n+3\cdot 2^n\cdot\left(3^2\right)^n-\left(3^3\right)^n$

$=8^n-3\cdot 4^n\cdot 3^n+3\cdot 2^n\cdot 9^n-27^n$

$=8^n-3\cdot(4\cdot 3)^n+3\cdot(2\cdot 9)^n-27^n$

$=8^n-3\cdot 12^n+3\cdot 18^n-27^n$

PRISJETIMO SE PRAVILA $(a^m)^n = a^{m \cdot n}$

DALJE KAKO JE $m \cdot n = n \cdot m$

TO JE $(a^m)^n = a^{m \cdot n} = a^{n \cdot m} = (a^n)^m$

TO SKRAĆENO PIŠEMO: $(a^m)^n = (a^n)^m$

TO SMO PRIMJENILI U 10) ZADATKU

① $(2^n)^2 = 2^{n \cdot 2} = 2^{2 \cdot n} = (2^2)^n = 4^n$

② $(3^n)^3 = 3^{n \cdot 3} = 3^{3 \cdot n} = (3^3)^n = 27^n$

5) $(2^n + 2^m)^3 = (2^n)^3 + 3 \cdot (2^n)^2 \cdot 2^m + 3 \cdot 2^n \cdot (2^m)^2 + (2^m)^3 =$

$= (2^3)^n + 3 \cdot 2^{2n} \cdot 2^m + 3 \cdot 2^n \cdot 2^{2m} + (2^3)^m =$

$= 8^n + 3 \cdot 2^{2n+m} + 3 \cdot 2^{n+2m} + 8^m$

PRIMJENILI SMO PRAVILO

$a^m \cdot a^n = a^{m+n}$

6) $(2^{n+1} - 2^{n-1})^3 = (2^{n+1})^3 - 3 \cdot (2^{n+1})^2 \cdot 2^{n-1} + 3 \cdot 2^{n+1} \cdot (2^{n-1})^2 - (2^{n-1})^3 =$

$= 2^{3 \cdot (n+1)} - 3 \cdot 2^{2(n+1)} \cdot 2^{n-1} + 3 \cdot 2^{n+1} \cdot 2^{2(n-1)} - 2^{3(n-1)} =$

$= 8^{n+1} - 3 \cdot 4^{n+1} \cdot 2^{n-1} + 3 \cdot 2^{n+1} \cdot 4^{n-1} - 8^{n-1}$

$= 8^n \cdot 8^1 - 3 \cdot 4^n \cdot 4^1 \cdot 2^n \cdot 2^{-1} + 3 \cdot 2^n \cdot 2^1 \cdot 4^n \cdot 4^{-1} - 8^n \cdot 8^{-1} =$

32. KAKO NAM ZADATAK KAŽE DA SE RADI O TREĆOJ POTENCIJI DVOČLANOG IZRAZA TO MOŽEMO RJEŠITI NA KRAĆI NAČIN (VIDEN U NEKIM GIMNAZIJAMA U ZGB.)

(I) KRAĆI NAČIN

ODAVDE UZIMAMO ZNAK

1) $a^3 + 6a^2 + 12a + 8 = (a+2)^3$

$A^3 = a^3 / \sqrt[3]{\ }$ $\qquad$ $B^3 = 8 / \sqrt[3]{\ }$

$A = \sqrt[3]{a^3}$ $\qquad$ $B = \sqrt[3]{8} = \sqrt[3]{2^3} = 2$

$A = a$ $\qquad$ $B = 2$

ILI ISPRAVNIJI DUŽI NAČIN:

(II)

$a^3 + 6a^2 + 12a + 8 = a^3 + 8 + 6a^2 + 12a =$

PREPOZNAJ A^3+B^3 $\quad = a^3 + 2^3 + 6a(a+2) =$

IZLUČIMO ZAJEDNIČKI FAKTOR $\rightarrow = (a+2)(a^2 - 2\cdot a + 2^2) + 6a(a+2)$ (Z.F.)

$= (a+2)(a^2 - 2a + 4 + 6a)$

$= (a+2)(a^2 + 4a + 4)$

$= (a+2)(a+2)^2 = (a+2)^{1+2} =$

$= (a+2)^3$

KAKO TEK KASNIJE RADITE RASTAVLJANJE NA FAKTORE TO BI I NAČIN BIO ONO ŠTO SU ONI OVDJE TRAŽILI ALI JA ĆU VAM OVO SVE RJEŠIT NA OBADVA NAČINA ....

32.

2) I

$27a^3 - 27a^2 + 9a - 1 =$ → ZNAK JE $(-)$

$A^3 = 27a^3 / \sqrt[3]{\ }$ $B = 1 / \sqrt[3]{\ }$

$A = \sqrt[3]{27a^3}$ $B = \sqrt[3]{1}$

$A = \sqrt[3]{27} \cdot \sqrt[3]{a^3}$ $B = 1$

$A = 3a$

PA JE $27a^3 - 27a^2 + 9a - 1 = (3a-1)^3$

II NAČIN

$27a^3 - 27a^2 + 9a - 1 = 27a^3 - 1 - 27a^2 + 9a =$

$= (3a)^3 - 1^3 - 9a(3a-1) =$

$= (3a-1)((3a)^2 + 3a \cdot 1 + 1^2) - 9a(3a-1) =$

$= (3a-1)(9a^2 + 3a + 1 - 9a) =$

$= (3a-1)(9a^2 - 6a + 1) =$

$= (3a-1)^1 \cdot (3a-1)^2 = (3a-1)^{1+2}$

$= (3a-1)^3$

---

3)

$a^3 - 21a^2 + 147a - 343 =$

$A^3 = a^3 / \sqrt[3]{\ }$ ZNAK $= (-)$

$A = a$ $B^3 = 343 / \sqrt[3]{\ }$

$B = \sqrt[3]{343}$

$B = 7$

PA JE

$a^3 - 21a^3 + 147a - 343 = (a-7)^3$

II NAČIN

$a^3 - 21a^2 + 147a - 343 =$

$= a^3 - 343 - 21a^2 + 147a =$

$= a^3 - 7^3 - 21a(a-7) =$

$= (a-7)(a^2 + a \cdot 7 + 7^2) - 21a(a-7) =$

$= (a-7)(a^2 + 7a + 49 - 21a) =$

$= (a-7)(a^2 - 14a + 49) =$

$= (a-7)^1 \cdot (a-7)^2 =$

$= (a-7)^3$

40.

Koristimo pravilo : $a^n b^n c^n = (abc)^n$

1.) $(2a-1)^2 \cdot (2a+1)^2 = ((2a-1)\cdot(2a+1))^2 = ((2a)^2 - 1^2)^2 = (2^2 a^2 - 1)^2 = (4a^2-1)^2 =$

$$= (4a^2)^2 - 2\cdot 4a^2 \cdot 1 + 1^2 = 4^2 \cdot (a^2)^2 - 8a^2 + 1 = 16a^4 - 8a^2 + 1$$

2.) $(4-4a+a^2)\cdot(4+4a+a^2) = \underbrace{(a^2+4}_{(A} - 4a)_{-B)} \cdot \underbrace{(a^2+4}_{(A} + 4a)_{+B)} = (a^2+4)^2 - (4a)^2 =$ $\quad = A^2 - B^2$

$$= (a^2)^2 + 2\cdot a^2 \cdot 4 + 4^2 - 4^2(a)^2 = a^4 + 8a^2 + 16 - 16a^2 =$$
$$= a^4 + 8a^2 - 16a^2 + 16 =$$
$$= a^4 - 8a^2 + 16 =$$
$$= (a^2)^2 - 2\cdot a^2 \cdot 4 + 4^2 =$$
$$= (a^2-4)^2$$

3.) $(a-1)^2 \cdot (a^2+1)^2 \cdot (a+1)^2 = ((a-1)\cdot(a^2+1)\cdot(a+1))^2 = ((a-1)\cdot(a+1)\cdot(a^2+1))^2 =$

$$= ((a^2-1)\cdot(a^2+1))^2 =$$
$$= ((a^2)^2 - 1^2)^2 =$$
$$= (a^4-1)^2 =$$
$$= (a^4)^2 - 2\cdot a^4 \cdot 1 + 1^2 =$$
$$= a^8 - 2a^4 + 1$$

4.) $(a^2+a+1)^2 \cdot (a^2-a+1)^2 = ((a^2+a+1)\cdot(a^2-a+1))^2 = ((a^2+1+a)\cdot(a^2+1-a))^2 =$

$$= ((a^2+1)^2 - a^2)^2 =$$
$$= ((a^2)^2 + 2\cdot a^2 \cdot 1 + 1^2 - a^2)^2 =$$
$$= (a^4 + 2a^2 - a^2 + 1)^2 =$$
$$= (a^4 + a^2 + 1)^2 =$$
$$= (a^4 + a^2 + 1)\cdot(a^4 + a^2 + 1) =$$
$$= a^4 \cdot a^4 + a^4 \cdot a^2 + a^4 \cdot 1 + a^2 \cdot a^4 + a^2 \cdot a^2 + a^2 \cdot 1 + 1\cdot a^4 + 1\cdot a^2 + 1\cdot 1 =$$
$$= a^8 + a^6 + a^4 + a^6 + a^4 + a^2 + a^4 + a^2 + 1 =$$
$$= a^8 + a^6 + a^6 + a^4 + a^4 + a^4 + a^2 + a^2 + 1 =$$
$$= a^8 + 2\cdot a^6 + 3\cdot a^4 + 2\cdot a^2 + 1$$

5.) $\left(2a^2-2a-1\right)^2\cdot\left(2a^2+2a+1\right)^2=\left(\left(2a^2-2a-1\right)\cdot\left(2a^2+2a+1\right)\right)^2=$

$=\left[\left(2a^2-\left(2a+1\right)\right)\cdot\left(2a^2+\left(2a+1\right)\right)\right]^2=$

$=\left[\left(2a^2\right)^2-\left(2a+1\right)^2\right]^2=$

$=\left[2^2\left(a^2\right)^2-\left(\left(2a\right)^2+2\cdot2a\cdot1+1^2\right)\right]^2=$

$=\left[4a^4-\left(4a^2+4a+1\right)\right]^2=$

$=\left[4a^4-4a^2-4a-1\right]^2=$

$=\left(4a^4-4a^2-4a-1\right)\cdot\left(4a^4-4a^2-4a-1\right)=$

$=4a^4\cdot4a^4+4a^4\cdot\left(-4a^2\right)+4a^4\cdot\left(-4a\right)+4a^4\cdot\left(-1\right)-4a^2\cdot4a^4-4a^2\cdot\left(-4a^2\right)-4a^2\cdot\left(-4a\right)+$

$-4a^2\cdot\left(-1\right)-4a\cdot4a^4-4a\cdot\left(-4a^2\right)-4a\cdot\left(-4a\right)-4a\cdot\left(-1\right)-1\cdot\left(4a^4-4a^2-4a-1\right)=$

$=16a^8-16a^6-16a^5-4a^4-16a^6+16a^4+16a^3+4a^2-16a^5+16a^3+16a^2+$

$+4a-4a^4+4a^2+4a+1=$

$=16a^8-16a^6-16a^6-16a^5-16a^5-4a^4+16a^4-4a^4+16a^3+16a^3+4a^2+16a^2+4a^2+$

$+4a+4a+1=$

$=16a^8-32a^6-32a^5+8a^4+32a^3+24a^2+8a+1$

41.

Koristimo pravilo : $a^nb^nc^n=\left(abc\right)^n$ i formulu za kub binoma: $\left(a-b\right)^3=a^3-3a^2b+3ab^2-b^3$ (7)

Pr imjeni gornje pravilo ↕ ↕ Prepoznaj razliku kvadrata ↓

1.) $\left(a-b\right)^3\cdot\left(a+b\right)^3=\left(\left(a-b\right)\cdot\left(a+b\right)\right)^3=\left(a^2-b^2\right)^3=\left(a^2\right)^3-3\cdot\left(a^2\right)^2\cdot b^2+3\cdot a^2\cdot\left(b^2\right)^2-\left(b^2\right)^3=$

$=a^6-3\cdot a^4\cdot b^2+3\cdot a^2\cdot b^4-b^6=$

$=a^6-3a^4b^2+3a^2b^4-b^6$

Primjeni formulu br. (7) ↓

$\left(a^n-b^n\right)^3\cdot\left(a^n+b^n\right)^3=\left(\left(a^n-b^n\right)\cdot\left(a^n+b^n\right)\right)^3=\left(\left(a^n\right)^2-\left(b^n\right)^2\right)^3=\left(a^{2n}-b^{2n}\right)^3=$

$=\left(a^{2n}\right)^3-3\cdot\left(a^{2n}\right)^2\cdot b^{2n}+3\cdot a^{2n}\cdot\left(b^{2n}\right)^2-\left(b^{2n}\right)^3=$

$=a^{2n\cdot3}-3\cdot a^{2n\cdot2}\cdot b^{2n}+3\cdot a^{2n}\cdot b^{2n\cdot2}-b^{2n\cdot3}=$

$=a^{6n}-3a^{4n}b^{2n}+3a^{2n}b^{4n}-b^{6n}$

2) $(a^2-1)^3\cdot(a^2+1)^3\cdot(a^4+1)^3=\left[(a^2-1)\cdot(a^2+1)\cdot(a^4+1)\right]^3=$ Unutar zagrade prepoznaj razliku kvadrata

$=\left[\left((a^2)^2-1^2\right)\cdot(a^4+1)\right]^3=$

$=\left[(a^4-1)\cdot(a^4+1)\right]^3=$ Opet razlika kvadrata...

$=\left[(a^4)^2-1^2\right]^3=$

$=(a^8-1)^3=$ Primjeni formulu br. (7)

$=(a^8)^3-3\cdot(a^8)^2\cdot1+3\cdot a^8\cdot1^2-1^3=$

$=a^{24}-3a^{16}+3a^8-1$

42.

Treba prepoznati da se radi o zbroju kubova: $A^3 + B^3 = (A+B)\cdot(A^2-A\cdot B+B^2)$ formula br. (9)

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

1.) $27a^3+8b^3=3^3\cdot a^3+2^3\cdot b^3=(3\cdot a)^3+(2\cdot b)^3=(3a+2b)\cdot\left((3a)^2-3a\cdot2b+(2b)^2\right)=$

$=(3a+2b)\cdot(9a^2-6ab+4b^2)$

Treba prepoznati da se radi o razlici kubova: $A^3 - B^3 = (A-B)\cdot(A^2+A\cdot B+B^2)$ formula br. (8)

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

2.) $1-64a^3=1^3-4^3\cdot a^3=1^3-(4\cdot a)^3=(1-4a)\cdot\left(1^2+1\cdot4a+(4a)^2\right)=$

$=(1-4a)\cdot(1+4a+16a^2)$

3.) $8a^3b^3+1=2^2a^3b^3+1^3=(2ab)^3+1^3=(2ab+1)\cdot\left((2ab)^2-2ab\cdot1+1^2\right)=$

$=(2ab+1)\cdot(4a^2b^2-2ab+1)$

4.) $125a^3-64b^6=5^3a^3-4^3(b^2)^3=(5a)^3-(4b^2)^3=(5a-4b^2)\cdot\left((5a)^2+5a\cdot4b^2+(4b^2)^2\right)=$

$=(5a-4b^2)\cdot(25a^2+20ab^2+16b^4)$

2.3. Rastavljanje na faktore

1.

1.) $2a^2+4ab^2=2\cdot a\cdot a\cdot b+2\cdot 2\cdot a\cdot b\cdot b=2\cdot a\cdot b\cdot(a+2b)$

Kako sam to napravio: 1. Svaki od članova rastavim na faktore...
2. Izlučim zajednički faktor...

Još jednom isti zadatak:

$$2a^2+4ab^2=2\cdot a\cdot a\cdot b+2\cdot 2\cdot a\cdot b\cdot b=$$

b^2 – rastavim na $b\cdot b$

a^2 – rastavim na: $a\cdot a$

–dalje - sada u izrazu: $=\underline{2}\cdot\underline{a}\cdot a\cdot\underline{b}+\underline{2}\cdot 2\cdot\underline{a}\cdot\underline{b}\cdot b=$ podvučem zajedničke faktore prvom i drugom članu te ih izlučimo:

$$=2\cdot a\cdot b\cdot\left(\frac{2\cdot a\cdot a\cdot b}{2\cdot a\cdot b}+\frac{2\cdot 2\cdot a\cdot b\cdot b}{2\cdot a\cdot b}\right)=$$ svaki od članova pisemo sada kao razlomak sa zajedničkim faktorom kao nazivnikom

nakon kračenja dobijemo: $=2ab\cdot(a+2b)$

2.) $3a^4b+15a^2b^2=\underline{3}\cdot\underline{a^2}\cdot a^2\cdot\underline{b}+5\cdot\underline{3}\cdot\underline{a^2}\cdot\underline{b}\cdot b=$

$$=3\cdot a^2\cdot b\cdot\left(\frac{3\cdot a^2\cdot a^2\cdot b}{3\cdot a^2\cdot b}+\frac{5\cdot 3\cdot a^2\cdot b\cdot b}{3\cdot a^2\cdot b}\right)= \textit{kratimo}$$

$$=3a^2b\cdot(a^2+5b)$$

uvjek sve članove unutar zagrade podjelimo sa zajedničkim faktorom kojeg smo izlučili, u ovom zadatku Z.F. $=3a^2b$

3) $6a^3b+8a^2b^3=2\cdot 3\cdot a\cdot a\cdot a\cdot b+2\cdot 4\cdot a\cdot a\cdot b\cdot b\cdot b$

$$=2\cdot a\cdot a\cdot b\cdot\left(\frac{2\cdot 3\cdot a\cdot a\cdot a\cdot b}{2\cdot a\cdot a\cdot b}+\frac{2\cdot 4\cdot a\cdot a\cdot b\cdot b\cdot b}{2\cdot a\cdot a\cdot b}\right)$$

$$=2a^2b\cdot(3a+4b^2)$$

4) $9a^4b^2 - 15a^2b^3 = 3\cdot3\cdot a\cdot a\cdot a\cdot a\cdot b\cdot b - 3\cdot5\cdot a\cdot a\cdot b\cdot b\cdot b =$

$= 3\cdot a\cdot a\cdot b\cdot b\left(\frac{3\cdot3\cdot a\cdot a\cdot a\cdot a\cdot b\cdot b}{3\cdot a\cdot a\cdot b\cdot b} - \frac{3\cdot5\cdot a\cdot a\cdot b\cdot b\cdot b}{3\cdot a\cdot a\cdot b\cdot b}\right)$

$= 3a^2b^2\cdot(3\cdot a\cdot a - 5\cdot b)$

$= 3a^2b^2\cdot(3a^2 - 5b)$

5) $10a^2b^3c + 5ab^2c^4 = 5\cdot2\cdot a\cdot a\cdot b\cdot b\cdot b\cdot c + 5\cdot a\cdot b\cdot b\cdot c\cdot c\cdot c\cdot c =$

$= 5\cdot a\cdot b\cdot b\cdot c\cdot\left(\frac{5\cdot2\cdot a\cdot a\cdot b\cdot b\cdot b\cdot c}{5\cdot a\cdot b\cdot b\cdot c} + \frac{5\cdot a\cdot b\cdot b\cdot c\cdot c\cdot c\cdot c}{5\cdot a\cdot b\cdot b\cdot c}\right)$

$= 5\cdot a\cdot b\cdot b\cdot c\cdot(2a\cdot b + 5\cdot c\cdot c\cdot c)$

$= 5ab^2c\cdot(2ab + 5c^3)$

6) $5a^3b^2 + 20a^2b^4 = \underline{5}\cdot\underline{a^2}\cdot a^1\cdot\underline{b^2} + \underline{5}\cdot4\cdot\underline{a^2}\cdot\underline{b^2}\cdot b^2 =$

$= 5\cdot a^2\cdot b^2\cdot\left(\frac{5\cdot a^2\cdot a^1\cdot b^2}{5\cdot a^2\cdot b^2} + \frac{5\cdot4\cdot a^2\cdot b^2\cdot b^2}{5\cdot a^2\cdot b^2}\right) =$

$= 5a^2b^2\cdot(a + 4b^2)$

2.

1) $6a^2b^2 - 12a^2b + 18ab^2 = 6\cdot a\cdot a\cdot b\cdot b - 2\cdot6a\cdot a\cdot b + 3\cdot6\cdot a\cdot b\cdot b =$

$= 6\cdot a\cdot b\cdot\left(\frac{6\cdot a\cdot a\cdot b\cdot b}{6\cdot a\cdot b} - \frac{2\cdot6\cdot a\cdot a\cdot b}{6\cdot a\cdot b} + \frac{3\cdot6\cdot a\cdot b\cdot b}{6\cdot a\cdot b}\right) =$

$= 6\cdot a\cdot b\cdot(a\cdot b - 2a + 3\cdot b) =$

$= 6ab(ab - 2a + 3b)$

2) $7a^3b+14a^2b^2-21a^2b=7\cdot a^2\cdot a^1\cdot b+7\cdot2\cdot a^2\cdot b\cdot b-7\cdot3\cdot a^2\cdot b=$

$$=7\cdot a^2\cdot b\cdot\left(\frac{7\cdot a^2\cdot a^1\cdot b}{7\cdot a^2\cdot b}+\frac{7\cdot2\cdot a^2\cdot b\cdot b}{7\cdot a^2\cdot b}-\frac{7\cdot3\cdot a^2\cdot b}{7\cdot a^2\cdot b}\right)=$$

$$=7a^2b\cdot(a+2b-3)$$

$\updownarrow$

uvjek sve članove unutar zagrade podjelimo sa zajedničkim faktorom kojeg smo izlučili

trebali bi znati da je razlomačka crta ustvari znak djeljenja

3) $10a^3b^2c-15a^2b^3c+25ab^3c^3=\underline{5}\cdot2\cdot\underline{a}\cdot a^2\cdot\underline{b^2}\cdot\underline{c}-\underline{5}\cdot3\cdot\underline{a}\cdot a\cdot\underline{b^2}\cdot b^1\cdot\underline{c}+\underline{5}\cdot5\cdot\underline{a}\cdot\underline{b^2}\cdot b\cdot\underline{c}\cdot c^2=$

$$=5\cdot a\cdot b^2\cdot c\cdot\left(\frac{5\cdot2\cdot a\cdot a^2\cdot b^2\cdot c}{5\cdot a\cdot b^2\cdot c}-\frac{5\cdot3\cdot a\cdot a\cdot b^2\cdot b^1\cdot c}{5\cdot a\cdot b^2\cdot c}+\frac{5\cdot5\cdot a\cdot b^2\cdot b\cdot c\cdot c^2}{5\cdot a\cdot b^2\cdot c}\right)=$$

$$=5ab^2c\cdot(2a^2-3ab+5bc^2)$$

Važno: Kako znamo na koje potencije rastavljamo faktore ?

Pogledajmo prvo za (a): prvi član ima a^3, drugi član ima a^2, treći član ima $a=a^1$ } najmanja potencija je a^1 pa stoga sve članove rastavimo na $a^1\cdot a^n\ldots$

Dakle: u prvom članu a^3 rastavimona $a^1\cdot a^2$

u drugom članu $a^2=a^1\cdot a^1$

u trečem članu a ostaje $a=a^1$

Za (b)

prvi član ima b^2, drugi član ima b^3, treći član ima b^3 } $\Rightarrow$ najmanja potencija je b^2 pa sve članove rastavimo na $b^2\cdot b^n$ } $\Rightarrow$ $b^2=b^2$, $b^3=b^2\cdot b^1$, $b^3=b^2\cdot b^1$

Za (c) prvi i drugi član imaju c^1, treći ima c^3 $\Rightarrow$ najmanja potencija je c^1....

4) $33a^4b^3c^2-44a^4bc^4+55a^3b^2c^4=$

$$=3\cdot\underline{11}\cdot\underline{a^3}\cdot a^1\cdot\underline{b^1}\cdot b^2\underline{c^2}-4\cdot\underline{11}\cdot\underline{a^3}\cdot a^1\cdot\underline{b}\cdot\underline{c^2}\cdot c^2+5\cdot\underline{11}\cdot\underline{a^3}\cdot\underline{b^1}\cdot b^1\cdot\underline{c^2}\cdot c^2=$$

$$=11\cdot a^3\cdot b^1\cdot c^2\cdot\left(\frac{3\cdot11\cdot a^3\cdot a^1\cdot b^1\cdot b^2c^2}{11\cdot a^3\cdot b^1\cdot c^2}-\frac{4\cdot11\cdot a^3\cdot a^1\cdot b\cdot c^2\cdot c^2}{11\cdot a^3\cdot b^1\cdot c^2}+\frac{5\cdot11\cdot a^3\cdot b^1\cdot b^1\cdot c^2\cdot c^2}{11\cdot a^3\cdot b^1\cdot c^2}\right)=$$

$$=11a^3bc^2\cdot(3ab^2-4ac^2+5bc^2)$$

5) $30a^3b^3c^2 - 18a^2b^4c^3 + 6a^2b^2c^2 =$

$$= 6\cdot5\cdot a^2\cdot a^1\cdot b^2\cdot b\cdot c^2 - 6\cdot3\cdot a^2\cdot b^2\cdot b^2\cdot c^2\cdot c^1 + 6\cdot a^2\cdot b^2\cdot c^2 =$$

$$= 6\cdot a^2\cdot b^2\cdot c^2\cdot\left(\frac{6\cdot5\cdot a^2\cdot a^1\cdot b^2\cdot b\cdot c^2}{6\cdot a^2\cdot b^2\cdot c^2} - \frac{6\cdot3\cdot a^2\cdot b^2\cdot b^2\cdot c^2\cdot c^1}{6\cdot a^2\cdot b^2\cdot c^2} + \frac{6\cdot a^2\cdot b^2\cdot c^2}{6\cdot a^2\cdot b^2\cdot c^2}\right) =$$

$$= 6a^2b^2c^2\cdot(5ab - 3b^2c + 1)$$

NAKON KRAĆENJA = 1
PAZI DOSTA VAS TU PIŠE DA JE TO NIŠTA ILI NEKA ŠTO NIJE TOČNO!

6) $27a^2b^4c - 36a^3b^4 - 63a^2b^3c^2 =$

$$= 3\cdot9\cdot a^2\cdot b^3\cdot b^1\cdot c - 4\cdot9\cdot a^2\cdot a^1\cdot b^1\cdot b^3 - 7\cdot9\cdot a^2\cdot b^3\cdot c^2 =$$

kratimo

$$= 9\cdot a^2\cdot b^3\cdot\left(\frac{3\cdot9\cdot a^2\cdot b^3\cdot b^1\cdot c}{9\cdot a^2\cdot b^3} - \frac{4\cdot9\cdot a^2\cdot a^1\cdot b^1\cdot b^3}{9\cdot a^2\cdot b^3} - \frac{7\cdot9\cdot a^2\cdot b^3\cdot c^2}{9\cdot a^2\cdot b^3}\right) =$$

$$= 9a^2b^3\cdot(3bc - 4ab - 7c^2)$$

Primjer:

$18x^3y^2z^2 + 12x^2y^2z^3 + 6x^3yz^3 =$

$$= 3\cdot6\cdot x^2\cdot x^1\cdot y^1\cdot y^1\cdot z^2 + 2\cdot6\cdot x^2\cdot y^1\cdot y^1\cdot z^2\cdot z^1 + 6\cdot x^2\cdot x^1\cdot y^1\cdot z^2\cdot z^1 =$$

$$= 6\cdot x^2\cdot y^1\cdot z^2\cdot\left(\frac{3\cdot6\cdot x^2\cdot x^1\cdot y^1\cdot y^1\cdot z^2}{6\cdot x^2\cdot y^1\cdot z^2} + \frac{2\cdot6\cdot x^2\cdot y^1\cdot y^1\cdot z^2\cdot z^1}{6\cdot x^2\cdot y^1\cdot z^2} + \frac{6\cdot x^2\cdot x^1\cdot y^1\cdot z^2\cdot z^1}{6\cdot x^2\cdot y^1\cdot z^2}\right) =$$

$$= 6\cdot x^2\cdot y\cdot z^2\cdot(3\cdot x\cdot y + 2\cdot y^1\cdot z^1 + x^1\cdot z^1) =$$

$$= 6x^2yz^2\cdot(3xy + 2yz + xz)$$

(*) U OVIM I SVIM SLIJEDEĆIM ZADACIMA KADA VAM NAPIŠEM KRATIMO UZMITE OLOVKU I POKRATITE ISTE ČLANOVE BROJNIKA I NAZIVNIKA, AKO BI JA TO RADIO ZADATAK BI BIO NEPREGLEDAN DAKLE OLOVKU U RUKE I KRIŽAJTE ...

8.

1.) $(ab-1)(a+2b)-(1-ab)(2a+b)=$

$\downarrow$ Treba primjetiti da je ovo ista prva zagrada samo sa suprotnim predznacima u tom slučaju izlučujemo uvijek (-1) iz te zagrade

$$(1-ab)=(-ab+1)=(-1\cdot ab-1\cdot 1)=-1\cdot(ab-1)$$

$$=(ab-1)(a+2b)-(-1)\cdot(ab-1)(2a+b)=$$
$$=(ab-1)(a+2b)+(ab-1)(2a+b)=$$
$$=\underline{(ab-1)}(a+2b)+\underline{(ab-1)}(2a+b)=$$
$$=(ab-1)\cdot\left(\frac{(ab-1)(a+2b)}{(ab-1)}+\frac{(ab-1)(2a+b)}{(ab-1)}\right)= \qquad (**)$$
$$=(ab-1)\cdot(a+2b\ +\ 2a+b)=$$
$$=(ab-1)\cdot(3a+3b)=(ab-1)\cdot 3\cdot(a+b)=3\cdot(ab-1)\cdot(a+b)$$

2.) $(2a-3)(b^2-2)+(2a+3)(2-b^2)=$

$\downarrow$

$$(2-b^2)=(-b^2+2)=(-1\cdot b^2-1\cdot(-2))=(-1)\cdot(b^2-2)$$

$$=(2a-3)(b^2-2)+(2a+3)\cdot(-1)\cdot(b^2-2)=$$
$$=(2a-3)(b^2-2)+(-1)\cdot(2a+3)\cdot(b^2-2)=$$
$$=(2a-3)\underline{(b^2-2)}-(2a+3)\underline{(b^2-2)}=(b^2-2)\cdot\left(\frac{(2a-3)(b^2-2)}{(b^2-2)}-\frac{(2a+3)(b^2-2)}{(b^2-2)}\right)=$$
$$=(b^2-2)\cdot((2a-3)-(2a+3))=$$
$$=(b^2-2)\cdot(2a-3-2a-3)=$$
$$=(b^2-2)\cdot(-6)=$$
$$=-6\cdot(b^2-2)$$

$\updownarrow$

Ovaj korak i korak u prethodnom zadatku označen sa $(**)$ obično radite u glavi ako ga ne pišete a rezultat vam ispada dobro meni ne smeta ali ja bih preporučio da se radi postupno...

13.

Kod ovog zadatka podrazumjevam da smo do sada svladali izlučivanje zajedničkog faktora Z. F. pa ću taj dio posla raditi malo skraćenim načinom ustvari onako kako to radite u školi

1.) $(2a-1)(a+2)^2-8a(2a-1)=$ → izlučimo Z. F. $(2a-1)$

$=(2a-1)\cdot\left((a+2)^2-8a\right)=$

$=(2a-1)\cdot\left(a^2+2\cdot a\cdot 2+2^2-8a\right)=$

$=(2a-1)\cdot\left(a^2+4a+4-8a\right)=$

$=(2a-1)\cdot\left(a^2-4a+4\right)=$

$=(2a-1)\cdot\left(a^2-2\cdot a\cdot 2+2^2\right)=$ → u drugoj zagradi treba prepoznati kvadrat razlike

$=(2a-1)\cdot(a-2)^2$

2.) $(a-2)(a-1)^2+4a(a-2)=$ → izlučimo Z. F. $(a-2)$

$=(a-2)\cdot\left[(a-1)^2+4a\right]=$

$=(a-2)\cdot\left(a^2-2a+1+4a\right)=$

$=(a-2)\cdot\left(a^2+2a+1\right)=$

$=(a-2)\cdot(a+1)^2$

$a^2+2a+1=(a+1)^2$

3.) $(a+3)(3a+1)^2-12a(a+3)=$

$=(a+3)\cdot\left[(3a+1)^2-12a\right]=$

$=(a+3)\cdot\left((3a)^2+2\cdot 3a\cdot 1+1^2-12a\right)=$

$=(a+3)\cdot\left(9a^2+6a-12a+1\right)=$

$=(a+3)\cdot\left(9a^2-6a+1\right)=$

$=(a+3)\cdot\left((3a)^2-2\cdot 3a\cdot 1+1^2\right)=$ → u drugoj zagradi treba prepoznati kvadrat razlike

$=(a+3)\cdot(3a-1)^2$

14.

$$A^2 \quad - \quad B^2 = (A-B)\cdot(A+B)$$

$$\updownarrow \qquad \updownarrow$$

→ prepoznaj razliku kvadrata i rastavi je na zaktore...

1.) $(a^2+b^2)^2 - 4a^2b^2 = (a^2+b^2)^2 - (2ab)^2 =$

$$= (a^2+b^2-2ab)\cdot(a^2+b^2+2ab) =$$

$$= (a^2-2ab+b^2)\cdot(a^2+2ab+b^2) =$$

$$\downarrow br.(3) \qquad \downarrow br.(1)$$

→ Prepoznaj formule br. (3) i (1)

$$= (a-b)^2 \quad \cdot \quad (a+b)^2 =$$

$$= (a-b)^2\cdot(a+b)^2$$

2.) $(a^2+1)^2 - 4a^2 = (a^2+1)^2 - 2^2a^2 = (a^2+1)^2 - (2a)^2 =$

$$= (a^2+1-2a)\cdot(a^2+1+2a) =$$

$$= (a^2-2a+1)\cdot(a^2+2a+1) =$$

$$= (a-1)^2\cdot(a+1)^2$$

3.) $(a^2+6ab)^2 - 81b^4 = (a^2+6ab)^2 - 9^2(b^2)^2 = (a^2+6ab)^2 - (9b^2)^2 =$

$$= (a^2+6ab-9b^2)\cdot(a^2+6ab+9b^2) =$$

$$= (a^2+6ab-9b^2)\cdot\left(a^2+2\cdot a\cdot 3b+(3b)^2\right) =$$

$$= (a^2+6ab-9b^2)\cdot(a+3b)^2 =$$

$$= (a+3b)^2\cdot(a^2+6ab-9b^2)$$

4.) $(a^2+4b^2)^2 - 16a^2b^2 = (a^2+4b^2)^2 - 4^2a^2b^2 = (a^2+4b^2)^2 - (4ab)^2 =$

$$= (a^2+4b^2-4ab)\cdot(a^2+4b^2+4ab) =$$

$$= (a^2-4ab+4b^2)\cdot(a^2+4ab+4b^2) =$$

$$= \left(a^2-2\cdot a\cdot 2b+(2b)^2\right)\cdot\left(a^2+2\cdot a\cdot 2b+(2b)^2\right) =$$

$$= (a-2b)^2\cdot(a+2b)^2$$

15.

1.) $a^2(b-1)-b^2(b-1)=a^2\underline{(b-1)}-b^2\underline{(b-1)}=$

$=(b-1)\cdot(a^2-b^2)=$

$=(b-1)\cdot(a-b)\cdot(a+b)$

2.) $x^2(x+y-1)-x-y+1=x^2(x+y-1)-1\cdot(x+y-1)=$

$=x^2\underline{(x+y-1)}-1\cdot\underline{(x+y-1)}=$ → podvuci Z.F. i izlući ga...

$=(x+y-1)\cdot(x^2-1)=$

$=(x+y-1)\cdot(x^2-1^2)=$ → druga zagrada je razlika kvadrata

$=(x+y-1)\cdot(x-1)\cdot(x+1)$

3.) $4a^2(x-1)-4x+4=4a^2(x-1)-4\cdot(x-1)=$

$=4a^2\underline{(x-1)}-4\cdot\underline{(x-1)}=$

$=(x-1)\cdot(4a^2-4)=(x-1)\cdot4\cdot(a^2-1)=$

$=4\cdot(x-1)\cdot(a^2-1^2)=$

$=4\cdot(x-1)\cdot(a-1)\cdot(a+1)$

Ovaj zadatak možemo riješiti i ovako:

$4a^2(x-1)-4x+4=4\cdot a^2(x-1)-4\cdot x+4\cdot1=4\cdot(a^2(x-1)-x+1)=$

$=4\cdot[a^2(x-1)-1\cdot(x-1)]=$

$=4\cdot(x-1)\cdot(a^2-1)=$

$=4\cdot(x-1)\cdot(a-1)\cdot(a+1)$

4.) $9a^2(b^2-1)-4b^2+4=9a^2(b^2-1)-4\cdot b^2+4\cdot1=$

$=9a^2(b^2-1)-4\cdot(b^2-1)=$

$=9a^2\underline{(b^2-1)}-4\cdot\underline{(b^2-1)}=$

$=(b^2-1)\cdot(9a^2-4)=$ → obadvije zagrade su razlike kvadrata br.(

$=(b^2-1^2)\cdot((3a)^2-2^2)=$

$=(b-1)\cdot(b+1)\cdot(3a-2)\cdot(3a+2)$

5.) $a^2-4b^2-9b^2(a^2-4b^2)=1\cdot(a^2-4b^2)-9b^2\cdot(a^2-4b^2)=$

$=1\cdot\underline{(a^2-4b^2)}-9b^2\cdot\underline{(a^2-4b^2)}=$ → podvučemo i zlučimo Z. F.

$=(a^2-4b^2)\cdot(1-9b^2)=$ → obadvije zagrade su razlike kvadrata br. (5)

$=\left(a^2-(2b)^2\right)\cdot\left(1^2-(3b)^2\right)=$ → rastavimo ih na faktore

$=(a-2b)\cdot(a+2b)\cdot(1-3b)\cdot(1+3b)$

6.) $a^2-1-ab+b=a^2-1^2-a\cdot b-b\cdot(-1)=$

$=(a-1)\cdot(a+1)-b\cdot(a-1)=$

$=\underline{(a-1)}\cdot(a+1)-b\cdot\underline{(a-1)}=$

$=(a-1)\cdot(a+1-b)=$

$=(a-1)\cdot(a-b+1)$

7.) $x^2-xy-y-1=x^2-1-xy-y=$ → promjenimo redosljed članova

$=x^2-1^2-y\cdot(x+1)=$

$=(x-1)\cdot(x+1)-y\cdot(x+1)=$

$=(x-1)\cdot\underline{(x+1)}-y\cdot\underline{(x+1)}=$

$=(x+1)\cdot(x-1-y)\cdot$

$=(x+1)\cdot(x-y-1)$

8.) $a^2b^2-a^2-ab^2+a=a^2\cdot(b^2-1)-a\cdot b^2-a\cdot(-1)=$

$=a^2\cdot(b^2-1)-a\cdot(b^2-1)=$

$=a\cdot\underline{a\cdot(b^2-1)}-1\cdot\underline{a\cdot(b^2-1)}=$

$=a\cdot(b^2-1)\cdot(a-1)=$

$=a\cdot(b-1)\cdot(b+1)\cdot(a-1)$

9.) $a^2b-a^2-b^2+1=a^2\cdot b-a^2\cdot 1-1\cdot b^2-1\cdot(-1)=$ → zapamti: $1=(-1)\cdot(-1)$

$=a^2\cdot(b-1)-1\cdot(b^2-1)=$ → druga zagrada je razlika kvadrata

$=a^2\cdot(b-1)-1\cdot(b-1)\cdot(b+1)=$

$a^2\cdot\underline{(b-1)}-1\cdot\underline{(b-1)}\cdot(b+1)=$

$=(b-1)\cdot\left(a^2-1\cdot(b+1)\right)=$

$=(b-1)\cdot(a^2-b-1)$

2.4. Algebarski razlomci

1.

1.) $$\frac{a^2-ab}{ab-b^2}=\frac{a\cdot a-a\cdot b}{a\cdot b-b\cdot b}=\frac{a\cdot(a-b)}{b\cdot(a-b)}=\left|\text{kratimo}\right|=\frac{a}{b}$$

ili kraće: $$\frac{a^2-ab}{ab-b^2}=\frac{a\cdot(a-b)}{b\cdot(a-b)}=\frac{a}{b}$$

2.) $$\frac{3a^2+3ab}{6ab+6b^2}=\frac{3\cdot a\cdot a+3\cdot a\cdot b}{6\cdot a\cdot b+6\cdot b\cdot b}=\frac{3\cdot a\cdot(a\cdot+b)}{6\cdot b\cdot(a+b)}=\frac{3\cdot a}{2\cdot 3\cdot b}=\frac{a}{2b}$$

3.) $$\frac{6a^2-9ab}{8a^2b-12ab^2}=\frac{3\cdot 2\cdot a\cdot a-3\cdot 3\cdot a\cdot b}{4\cdot 2\cdot a\cdot a\cdot b-4\cdot 3\cdot a\cdot b\cdot b}=\left|\begin{array}{c}\text{izlučimo zajednički}\\ \text{faktor u brojniku i}\\ \text{nazivniku}\end{array}\right|=\frac{3\cdot a\cdot(2\cdot a-3\cdot b)}{4\cdot a\cdot b\cdot(2\cdot a-3\cdot b)}\overset{\text{kratimo}}{=}\frac{3}{4b}$$

4.) $$\frac{15a^2-10ab}{3ab-2b^2}=\frac{3\cdot 5\cdot a\cdot a-5\cdot 2\cdot a\cdot b}{3\cdot a\cdot b-2\cdot b\cdot b}=\frac{5\cdot a\cdot(3\cdot a-2\cdot b)}{b\cdot(3\cdot a-2\cdot b)}=\frac{5\cdot a}{b}=\frac{5a}{b}$$

5.) $$\frac{24a^2b-36ab^2}{18a^2-12ab}=\frac{6\cdot 4\cdot a\cdot a\cdot b-6\cdot 6\cdot a\cdot b\cdot b}{6\cdot 3\cdot a\cdot a-6\cdot 2\cdot a\cdot b}=\frac{6\cdot a\cdot b\cdot(4\cdot a-6\cdot b)}{6\cdot a\cdot(3\cdot a-2\cdot b)}=\frac{b\cdot(4a-6b)}{3a-2b}=$$

$$=\frac{b\cdot(2\cdot 2\cdot a-2\cdot 3\cdot b)}{3a-2b}=\frac{b\cdot 2\cdot(2\cdot a-3\cdot b)}{3a-2b}=\frac{2b\cdot(2a-3b)}{3a-2b}$$

6.) $$\frac{12a^2b+4ab^2}{24a^3b^2+8a^2b^3}=\frac{4\cdot 3\cdot a\cdot a\cdot b+4\cdot a\cdot b\cdot b}{8\cdot 3\cdot a^2\cdot a\cdot b^2+8\cdot a^2\cdot b^2\cdot b^1}=\frac{4\cdot a\cdot b\cdot(3\cdot a+b)}{8\cdot a^2\cdot b^2\cdot(3\cdot a+b)}=\frac{4\cdot a\cdot b}{8\cdot a^2\cdot b^2}=$$

$$=\frac{4\cdot a\cdot b}{4\cdot 2\cdot a\cdot a\cdot b\cdot b}=\frac{1}{2\cdot a\cdot b}=\frac{1}{2ab}$$

2.

1) $\frac{2a^2b-2ab^2}{4a^2b+4ab^2}=\frac{2\cdot a\cdot a\cdot b-2\cdot a\cdot b\cdot b}{4\cdot a\cdot a\cdot b+4\cdot a\cdot b\cdot b}$

$=\frac{2ab(a-b)}{4ab(a+b)}$ ⇒ izlučili smo zajednički faktor iz brojnika i nazivnika i pokratili

$=\frac{a-b}{2(a+b)}$

2) $\frac{a^3+a^2b}{ab^2+b^3}=\frac{a^2(a+b)}{b^2(a+b)}$ ⇒ izlučimo zajedničke faktore u brojniku i nazivniku i pokratimo

$=\frac{a^2}{b^2}=\left(\frac{a}{b}\right)^2$

3) $\frac{a^4b^2-2a^3b^3}{2a^3b^2+a^2b^3}=\frac{a^3b^2(a-2b)}{a^2b^2(2a+b)}$ ISTO KAO I U PRETHODNIM ZADACIMA

$=\frac{a(a-2b)}{(2a+b)}$

4) $\frac{3a^3b-3a^2b^2}{3ab^3-3a^2b^2}=\frac{3a^2b(a-b)}{3ab^2(b-a)}$ ⇒ u brojniku izlučimo -1 iz $(a-b)$

$=\frac{-1\cdot 3a^2b(b-a)}{3ab^2(b-a)}$ ⇒ pokratimo zajedničke članove iz brojnika i nazivnika

$=-\frac{a}{b}$

5) $\frac{2a^3b-2ab^2}{4a^2b-4a^4}=\frac{2ab(a^2-b)}{4a^2(b-a^2)}$ ⇒ izlučimo -1 iz (a^2-b) u brojniku

$=\frac{-1\cdot 2ab(b-a^2)}{4a^2(b-a^2)}$ ⇒ kratimo

$=-\frac{b}{2a}$

6) $\frac{2a^2b^2c^2-ac^3}{bc^2-2ab^3c}=\frac{ac^2(2ab^2-c)}{bc(c-2ab^2)}$ ⇒ izlučimo -1 iz brojnika

$=\frac{-1\cdot ac^2(c-2ab^2)}{bc(c-2ab^2)}$ ⇒ kratimo

$=-\frac{ac}{b}$

3.

Prepoznaj razliku kvadrata i rastavi je na faktore
↓ ↓ ↓

1.) $$\frac{a^2-16}{3a^2+12a}=\frac{a^2-4^2}{3\cdot a\cdot a+3\cdot 4\cdot a}=\frac{(a-4)\cdot(a+4)}{3\cdot a\cdot(a+4)}=\frac{a-4}{3a}$$

Prepoznaj razliku kvadrata i rastavi je na faktore
↓ ↓ ↓

2.) $$\frac{2a^2-18}{4a^2+12a}=\frac{2\cdot a^2-2\cdot 9}{4\cdot a\cdot a+4\cdot 3\cdot a}=\frac{2\cdot(a^2-9)}{4\cdot a\cdot(a+3)}=\frac{2\cdot(a^2-3^2)}{2\cdot 2\cdot a\cdot(a+3)}=\frac{(a-3)(a+3)}{2\cdot a\cdot(a+3)}=\frac{a-3}{2a}$$

3.) $$\frac{3a^2-27}{4a^2-12a}=\frac{3\cdot a^2-3\cdot 9}{4\cdot a\cdot a-4\cdot 3\cdot a}=\frac{3\cdot(a^2-9)}{4\cdot a\cdot(a-3)}=\frac{3\cdot(a^2-3^2)}{4\cdot a\cdot(a-3)}=\frac{3\cdot(a-3)\cdot(a+3)}{4\cdot a\cdot(a-3)}=\frac{3\cdot(a+3)}{4a}$$

4.) $$\frac{a^3-25a}{2a^2-10a}=\frac{a^1\cdot a^2-25\cdot a}{2\cdot a\cdot a-2\cdot 5\cdot a}=\frac{a\cdot(a^2-25)}{2\cdot a\cdot(a-5)}=\frac{(a^2-5^2)}{2\cdot(a-5)}=\frac{(a-5)\cdot(a+5)}{2\cdot(a-5)}=\frac{a+5}{2}$$

5) $$\frac{8-2a^2}{2a^2-4a}=\frac{2\cdot 4-2\cdot a^2}{2\cdot a\cdot a-2\cdot 2\cdot a}=\frac{2\cdot(4-a^2)}{2\cdot a\cdot(a-2)}=\frac{2\cdot(2^2-a^2)}{2\cdot a\cdot(a-2)}=\frac{2\cdot(2-a)\cdot(2+a)}{2\cdot a\cdot(a-2)}=\frac{(2-a)\cdot(2+a)}{a\cdot(a-2)}=$$

$$=\frac{-1\cdot(a-2)\cdot(2+a)}{a\cdot(a-2)}=\frac{-1\cdot(a+2)}{a}=-1\cdot\frac{(a+2)}{a}=-1\cdot\frac{a+2}{a}=-\frac{a+2}{a}$$

6) $$\frac{4a^2-9b^2}{6ab^2-4a^2b}=\frac{2^2a^2-3^2b^2}{2\cdot 3\cdot a\cdot b\cdot b-2\cdot 2\cdot a\cdot a\cdot b}=\frac{(2a)^2-(3b)^2}{2\cdot a\cdot b\cdot(3\cdot b-2\cdot a)}=\frac{(2a-3b)\cdot(2a+3b)}{2ab\cdot(3b-2a)}=$$

↓

$$2ab\cdot(3b-2a)=2ab\cdot((-1)\cdot(2a-3b))=-2ab\cdot(2a-3b)$$

$$=\frac{(2a-3b)\cdot(2a+3b)}{-2ab\cdot(2a-3b)}=-\frac{2a+3b}{2ab}$$

7) $$\frac{a^4+4a^2b^2}{a^5-16ab^4}=\frac{a^2\cdot a^2+4\cdot a^2\cdot b^2}{a^1\cdot a^4-16\cdot a\cdot b^4}=\frac{a^2\cdot\left(a^2+4\cdot b^2\right)}{a^1\cdot\left(a^4-16\cdot b^4\right)}=\frac{a\cdot a\cdot\left(a^2+4b^2\right)}{a\cdot\left(\left(a^2\right)^2-4^2\cdot\left(b^2\right)^2\right)}=$$

$$=\frac{a\cdot\left(a^2+4b^2\right)}{\left(a^2\right)^2-\left(4\cdot b^2\right)^2}=\frac{a\cdot\left(a^2+4b^2\right)}{\left(a^2-4\cdot b^2\right)\cdot\left(a^2+4\cdot b^2\right)}=\frac{a}{a^2-4b^2}$$

kratimo

8) $$\frac{4a^3-9a}{6a^2-9a}=\frac{4\cdot a^2\cdot a^1-9\cdot a^1}{3\cdot 2\cdot a\cdot a-3\cdot 3\cdot a}=\frac{a^1\cdot\left(4\cdot a^2-9\right)}{3\cdot a\cdot\left(2\cdot a-3\right)}=\frac{\left(2^2\cdot a^2-3^2\right)}{3\cdot\left(2a-3\right)}=\frac{\left(2\cdot a\right)^2-3^2}{3\cdot\left(2a-3\right)}=$$

$$=\frac{\left(2a-3\right)\cdot\left(2a+3\right)}{3\cdot\left(2a-3\right)}=\frac{2a+3}{3}$$

4.

Treba prepoznati da se radi o kvadratu razlike

$$A^2-2AB+B^2$$
$$\downarrow\quad\downarrow\quad\downarrow$$

1.) $$\frac{x^2-2x+1}{2x^2-2x}=\frac{\left(x-1\right)^2}{2\cdot x\cdot x-2\cdot x}=\frac{\left(x-1\right)\cdot\left(x-1\right)}{2\cdot x\cdot\left(x-1\right)}=\frac{x-1}{2x}$$

2.) $$\frac{a^2-2a}{a^3-4a^2+4a}=\frac{a\cdot a-2\cdot a}{a^1\cdot a^2-4\cdot a^1\cdot a^1+4\cdot a}=\frac{a\cdot\left(a-2\right)}{a^1\cdot\left(a^2-4a+4\right)}=\frac{a\cdot\left(a-2\right)}{a\cdot\left(a-2\right)^2}=$$

$$=\frac{a\cdot\left(a-2\right)}{a\cdot\left(a-2\right)\cdot\left(a-2\right)}=\frac{1}{a-2}$$

↓ prepoznaj

$$a^2-4a+4=a^2-2\cdot 2\cdot a+2^2=\left(a-2\right)^2$$

3.) $$\frac{a^2+6a+9}{a^2-9}=\frac{a^2+2\cdot 3\cdot a+3^2}{a^2-3^2}=\frac{\left(a+3\right)^2}{\left(a-3\right)\cdot\left(a+3\right)}=\frac{\left(a+3\right)\cdot\left(a+3\right)}{\left(a-3\right)\cdot\left(a+3\right)}=\frac{a+3}{a-3}$$

$$\downarrow\quad\downarrow\qquad\uparrow\quad\uparrow$$

$$A^2-B^2=\left(A-B\right)\cdot\left(A+B\right)$$ – treba prepoznati razliku kvadrata u nazivniku te kvadrat zbroja u brojniku...

4.) $$\frac{a^2-2a+1}{1-a^2}=\frac{(a-1)^2}{(1-a)\cdot(1+a)}=\frac{(a-1)\cdot(a-1)}{-1\cdot(a-1)\cdot(a+1)}=\frac{a-1}{-1\cdot(a+1)}=-1\cdot\frac{a-1}{a+1}=\frac{-1\cdot(a-1)}{a+1}=$$
$$=\frac{-a+1}{a+1}=\frac{1-a}{1+a}$$

Dodatna uputa : u drugom koraku u nazivniku imamo izraz : $(1-a)\cdot(1+a)$ da bi skratili razlomak moramo prikaztati prvu zagradu sa predznakom minus jer tada članovi unutar zagrade mijenjaju svoj predznak i zagrada postaje identična onoj iz brojnika sa kojom je skratimo.

Dakle: $(1-a)=-1\cdot(-1+a)=-1\cdot(a-1)$

5.) $$\frac{x^2-5x}{(5-x)^2}=\frac{x\cdot x-5\cdot x}{(x-5)^2}=\frac{x\cdot(x-5)}{(x-5)\cdot(x-5)}=\frac{x}{x-5}$$

↓

Primjeni pravilo koje sam dao u formulama : $(a-b)^2=(b-a)^2$

6.) $$\frac{(2x-3)^2}{6x-4x^2}=\frac{(2x-3)^2}{-4x^2+6x}=\frac{(2x-3)^2}{-2\cdot 2\cdot x\cdot x+2\cdot 3\cdot x}=\frac{(2x-3)^2}{-2\cdot x\cdot(2x-3)}=\frac{(2x-3)\cdot(2x-3)}{-2x\cdot(2x-3)}=$$
$$=\frac{2x-3}{-2x}=-\frac{2x-3}{-1\cdot 2x}=-1\cdot\frac{2x-3}{2x}=\frac{-1\cdot(2x-3)}{2x}=\frac{-2x+3}{2x}=\frac{3-2x}{2x}$$

5.

Koristimo ove formule:

ALGEBARSKI IZRAZI	Br.
$(a+b)^2=(a+b)\cdot(a+b)=a^2+2ab+b^2$	(1)
$(a+b)^2=(b+a)^2$	(2)
$(a-b)^2=(a-b)\cdot(a-b)=a^2-2ab+b^2$	(3)
$(a-b)^2=(b-a)^2$	(4)
$(a-b)\cdot(a+b)=a^2-b^2$	(5)
$(a+b)^3=a^3+3a^2b+3ab^2+b^3$	(6)
$(a-b)^3=a^3-3a^2b+3ab^2-b^3$	(7)
$a^3-b^3=(a-b)\cdot(a^2+ab+b^2)$	(8)
$a^3+b^3=(a+b)\cdot(a^2-ab+b^2)$	(9)
$(a+b+c)^2=a^2+b^2+c^2+2ab+2ac+2bc$	(10)

1.) $$\frac{a-1}{1-a^2}=\frac{-1+a}{(1-a)\cdot(1+a)}=\frac{-1\cdot(1-a)}{(1-a)\cdot(1+a)}=\frac{-1}{1+a}=-\frac{1}{a+1}$$

2.) $\dfrac{a-1}{(1-a)^2} =$ | **U formulama sam dao pravilo br. (3) da je:** $(a-b)^2 = (b-a)^2$ **primjenimo ga ovdje:** | $= \dfrac{a-1}{(a-1)^2} = \dfrac{(a-1)}{(a-1)\cdot(a-1)} = \dfrac{1}{a-1}$

Po pravilu br. (3) ispada da je: $(1-a)^2 = (a-1)^2$ **to možete provjeriti kvadriranjem ...**

3.) $\dfrac{a^2-1}{(a-1)^2} = \dfrac{(a-1)\cdot(a+1)}{(a-1)\cdot(a-1)} = \dfrac{a+1}{a-1}$

Primjenio sam treće pravilo koje sam dao u formulama:

$(a-b)^2 = (a-b)\cdot(a-b) = \ldots$

4.) $\dfrac{(a+1)^2}{a^2-1} = \dfrac{(a+1)\cdot(a+1)}{(a-1)\cdot(a+1)} = \dfrac{a+1}{a-1}$

↓

Po br. (5) $a^2-b^2 = (a-b)\cdot(a+b)$

↑

5.) $\dfrac{a^2-1}{1-a^4} = \dfrac{(a-1)\cdot(a+1)}{1^2-(a^2)^2} = \dfrac{(-1+a)\cdot(a+1)}{(1-a^2)\cdot(1+a^2)} = \dfrac{-1\cdot(1-a)\cdot(a+1)}{(1-a)\cdot(a+1)\cdot(1+a^2)} = \dfrac{-1}{(a^2+1)} = -\dfrac{1}{a^2+1}$

6.) $\dfrac{a^4-1}{(1-a^2)^2} = \dfrac{(a^2)^2-1}{(a^2-1)^2} = \dfrac{(a^2)^2-1^2}{(a^2-1)\cdot(a^2-1)} = \dfrac{(a^2-1)\cdot(a^2+1)}{(a^2-1)\cdot(a^2-1)} = \dfrac{a^2+1}{a^2-1}$

↓

Po pravilu: $(a-b)^2 = (b-a)^2$